

# Advanced Data Management (CSCI 640/490)

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## Structured Data

Dr. David Koop

# Relational Algebra

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- Definition: A procedural language consisting of a set of operations that take one or two relations as input and produce a new relation as their result.
- Six basic operators
  - select:  $\sigma$
  - project:  $\Pi$
  - union:  $\cup$
  - set difference:  $-$
  - Cartesian product:  $\times$
  - rename:  $\rho$

[A. Silberschatz et al.]

# Select Operation

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- The select operation selects tuples that satisfy a given predicate.
- Notation:  $\sigma_p(r)$
- $p$  is called the selection predicate
- Example: select those tuples of the `instructor` relation where the instructor is in the “Physics” department.
  - Query:  $\sigma_{\text{dept\_name}=\text{“Physics”}}(\text{instructor})$

- Result:

<i>ID</i>	<i>name</i>	<i>dept_name</i>	<i>salary</i>
22222	Einstein	Physics	95000
33456	Gold	Physics	87000

# Project Operation

<i>ID</i>	<i>name</i>	<i>salary</i>
10101	Srinivasan	65000
12121	Wu	90000
15151	Mozart	40000
22222	Einstein	95000
32343	El Said	60000
33456	Gold	87000
45565	Katz	75000
58583	Califieri	62000
76543	Singh	80000
76766	Crick	72000
83821	Brandt	92000
98345	Kim	80000

- Example: eliminate the dept\_name attribute of instructor
- Query:  $\Pi_{ID, name, salary}(instructor)$

[A. Silberschatz et al.]

# Cartesian-Product Operation

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- The **Cartesian-product** operation (denoted by  $\times$ ) allows us to combine information from any two relations.
- Example: the Cartesian product of the relations `instructor` and `teaches` is written as: `instructor  $\times$  teaches`
- We construct a tuple of the result out of **each possible pair** of tuples: one from the `instructor` relation and one from the `teaches` relation
- Since the `instructor ID` appears in both relations we distinguish between these attribute by attaching to the attribute the name of the relation from which the attribute originally came.
  - `instructor.ID` and `teaches.ID`

[A. Silberschatz et al.]

# Join Operation

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- The Cartesian-Product `instructor X teaches` associates every tuple of `instructor` with every tuple of `teaches`.
  - Most of the resulting rows have information about instructors who **did not** teach a particular course.
- To get only those tuples of `instructor X teaches` that pertain to instructors and the courses that they taught, we write:

$\sigma_{\text{instructor.id} = \text{teaches.id}} (\text{instructor X teaches})$

- We get only those tuples of `instructor X teaches` that pertain to instructors and the courses that they taught.

[A. Silberschatz et al.]



# Equivalent Queries

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- Example: Find information about courses taught by instructors in the Physics department
- Query 1:  
$$\sigma_{\text{dept\_name}=\text{"Physics"}} (\text{instructor} \bowtie \text{teaches})$$
- Query 2  
$$(\sigma_{\text{dept\_name}=\text{"Physics"}} (\text{instructor})) \bowtie \text{teaches}$$
- The **order** of joins is one focus of some of the work on query optimization

[A. Silberschatz et al.]

# Components of SQL

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- **Data Definition Language (DDL)**: the specification of information about relations, including schema, types, integrity constraints, indices, storage
- **Data Manipulation Language (DML)**: provides the ability to query information from the database and to insert tuples into, delete tuples from, and modify tuples in the database.
- **Integrity**: the DDL includes commands for specifying integrity constraints.
- **View definition**: The DDL includes commands for defining views.
- Also: **Transaction control, embedded and dynamic SQL, authorization**

[A. Silberschatz et al.]



# Create Table

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- An SQL relation is defined using the create table command:

```
create table r (A1 D1, A2 D2, ..., An Dn, (C1), ..., (Ck))
```

- $r$  is the **name** of the relation
  - each  $A_i$  is an **attribute name** in the schema of relation  $r$
  - $D_i$  is the **data type** of values in the domain of attribute  $A_i$
- Example:

```
create table instructor(  
    ID          char(5),  
    name        varchar(20),  
    dept_name   varchar(20),  
    salary      numeric(8,2));
```

$C_i$  are integrity  
constraints:  
keys, foreign keys

[A. Silberschatz et al.]

# Basic Query Structure

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- A typical SQL query has the form:

**select**  $A_1, A_2, \dots, A_n$

**from**  $r_1, r_2, \dots, r_m$

**where**  $P$

- $A_i$  represents an **attribute**
  - $r_i$  represents a **relation**
  - $P$  is a **predicate**.
- The result of an SQL query is a **relation**

[A. Silberschatz et al.]

# Select

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- The **select** clause lists the attributes desired in the result of a query
  - corresponds to the projection operation of the relational algebra
- Example: Find the names of all instructors
  - **select** name  
**from** instructor;
- Note: SQL names are **case insensitive**
  - Name and NAME and name are equivalent
  - Some people use upper case for language keywords (e.g. SELECT)

[A. Silberschatz et al.]

# Where

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- The operands can be expressions with operators `<`, `<=`, `>`, `>=`, `=`, and `<>`
- SQL allows the use of the logical connectives `and`, `or`, and `not`
- Comparisons can be applied to results of arithmetic expressions
- Example: Find all instructors in Comp. Sci. with salary `> 70000`

```
- select name  
  from instructor  
  where dept_name = 'Comp. Sci.' and salary > 70000
```

<i>name</i>
Katz
Brandt

[A. Silberschatz et al.]

# From

---

- The **from** clause lists the relations involved in the query
  - Corresponds to the **Cartesian Product** operation in relational algebra
- Find the Cartesian product `instructor X teaches`
  - **select** \*
  - **from** `instructor, teaches;`
  - All possible `instructor – teaches` pair, with all attributes from both
  - Shared attributes (e.g., `ID`) are renamed (e.g., `instructor.ID`)
- Not very useful directly but useful combined with where clauses.

# Group By

- Find the average salary of instructors in each department
  - select** dept\_name, **avg**(salary) **as** avg\_salary
  - from** instructor
  - group by** dept\_name;

<i>ID</i>	<i>name</i>	<i>dept_name</i>	<i>salary</i>
76766	Crick	Biology	72000
45565	Katz	Comp. Sci.	75000
10101	Srinivasan	Comp. Sci.	65000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
22222	Einstein	Physics	95000

<i>dept_name</i>	<i>avg_salary</i>
Biology	72000
Comp. Sci.	77333
Elec. Eng.	80000
Finance	85000
History	61000
Music	40000
Physics	91000

[A. Silberschatz et al.]



# Deletion

---

- Delete all instructors: **delete from** instructor;
- Delete all instructors from the Finance department
  - **delete from** instructor  
**where** dept\_name= 'Finance';
- Delete all tuples in the instructor relation for those instructors associated with a department located in the Watson building
  - **delete from** instructor  
**where** dept\_name **in** (**select** dept\_name  
**from** department  
**where** building = 'Watson');

[A. Silberschatz et al.]

# Insertion

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- Make each student in the Music department who has earned more than 144 credit hours an instructor in the Music department with a salary of \$18,000.

```
- insert into instructor  
    select ID, name, dept_name, 18000  
    from student  
    where dept_name = 'Music' and total_cred > 144;
```

- The select-from-where statement is evaluated fully before any of its results are inserted into the relation.
- If not queries like

```
insert into table1 select * from table1
```

would cause problems

[A. Silberschatz et al.]

# Updates

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- Give a 5% salary raise to all instructors
  - **update** instructor  
  **set** salary = salary \* 1.05
- Give a 5% salary raise to those instructors who earn less than 70000
  - **update** instructor  
  **set** salary = salary \* 1.05  
  **where** salary < 70000;
- Give a 5% salary raise to instructors whose salary is less than average
  - **update** instructor  
  **set** salary = salary \* 1.05  
  **where** salary < (**select avg**(salary) **from** instructor);

[A. Silberschatz et al.]

# Assignment 2

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- Same questions as Assignment 1 but using pandas, duckdb, and ibis

# Joins

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- Join operations take two relations and return another relation.
- From relational algebra, this is a Cartesian product + selection
- Want tuples in the two relations to match (under some condition)
- The join operations typically used as subquery expressions in the from clause
- Three types of joins:
  - Natural join
  - Inner join
  - Outer join

[A. Silberschatz et al.]

# Natural Join

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- Natural join matches tuples with the same values for all **common** attributes, and retains only **one copy** of each common column.
- List the names of instructors along with the course ID of the courses that they taught
  - **select** name, course\_id  
**from** students, takes  
**where** student.ID = takes.ID;
- Same query in SQL with “natural join” construct
  - **select** name, course\_id  
**from** student **natural join** takes;

[A. Silberschatz et al.]



# Example: Student Schedules

<i>ID</i>	<i>name</i>	<i>dept_name</i>	<i>tot_cred</i>
00128	Zhang	Comp. Sci.	102
12345	Shankar	Comp. Sci.	32
19991	Brandt	History	80
23121	Chavez	Finance	110
44553	Peltier	Physics	56
45678	Levy	Physics	46
54321	Williams	Comp. Sci.	54
55739	Sanchez	Music	38
70557	Snow	Physics	0
76543	Brown	Comp. Sci.	58
76653	Aoi	Elec. Eng.	60
98765	Bourikas	Elec. Eng.	98
98988	Tanaka	Biology	120

<i>ID</i>	<i>course_id</i>	<i>sec_id</i>	<i>semester</i>	<i>year</i>	<i>grade</i>
00128	CS-101	1	Fall	2017	A
00128	CS-347	1	Fall	2017	A-
12345	CS-101	1	Fall	2017	C
12345	CS-190	2	Spring	2017	A
12345	CS-315	1	Spring	2018	A
12345	CS-347	1	Fall	2017	A
19991	HIS-351	1	Spring	2018	B
23121	FIN-201	1	Spring	2018	C+
44553	PHY-101	1	Fall	2017	B-
45678	CS-101	1	Fall	2017	F
45678	CS-101	1	Spring	2018	B+
45678	CS-319	1	Spring	2018	B
54321	CS-101	1	Fall	2017	A-
54321	CS-190	2	Spring	2017	B+
55739	MU-199	1	Spring	2018	A-
76543	CS-101	1	Fall	2017	A
76543	CS-319	2	Spring	2018	A
76653	EE-181	1	Spring	2017	C
98765	CS-101	1	Fall	2017	C-
98765	CS-315	1	Spring	2018	B
98988	BIO-101	1	Summer	2017	A
98988	BIO-301	1	Summer	2018	<i>null</i>

[A. Silberschatz et al.]



# Example: Natural Join

<i>ID</i>	<i>name</i>	<i>dept_name</i>	<i>tot_cred</i>	<i>course_id</i>	<i>sec_id</i>	<i>semester</i>	<i>year</i>	<i>grade</i>
00128	Zhang	Comp. Sci.	102	CS-101	1	Fall	2017	A
00128	Zhang	Comp. Sci.	102	CS-347	1	Fall	2017	A-
12345	Shankar	Comp. Sci.	32	CS-101	1	Fall	2017	C
12345	Shankar	Comp. Sci.	32	CS-190	2	Spring	2017	A
12345	Shankar	Comp. Sci.	32	CS-315	1	Spring	2018	A
12345	Shankar	Comp. Sci.	32	CS-347	1	Fall	2017	A
19991	Brandt	History	80	HIS-351	1	Spring	2018	B
23121	Chavez	Finance	110	FIN-201	1	Spring	2018	C+
44553	Peltier	Physics	56	PHY-101	1	Fall	2017	B-
45678	Levy	Physics	46	CS-101	1	Fall	2017	F
45678	Levy	Physics	46	CS-101	1	Spring	2018	B+
45678	Levy	Physics	46	CS-319	1	Spring	2018	B
54321	Williams	Comp. Sci.	54	CS-101	1	Fall	2017	A-
54321	Williams	Comp. Sci.	54	CS-190	2	Spring	2017	B+
55739	Sanchez	Music	38	MU-199	1	Spring	2018	A-
76543	Brown	Comp. Sci.	58	CS-101	1	Fall	2017	A
76543	Brown	Comp. Sci.	58	CS-319	2	Spring	2018	A
76653	Aoi	Elec. Eng.	60	EE-181	1	Spring	2017	C
98765	Bourikas	Elec. Eng.	98	CS-101	1	Fall	2017	C-
98765	Bourikas	Elec. Eng.	98	CS-315	1	Spring	2018	B
98988	Tanaka	Biology	120	BIO-101	1	Summer	2017	A
98988	Tanaka	Biology	120	BIO-301	1	Summer	2018	<i>null</i>

[A. Silberschatz et al.]

# Natural Join Danger

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- Beware of unrelated attributes with same name which get equated incorrectly
- Example: List the names of students instructors along with the titles of courses that they have taken
  - **select** name, title  
**from** student **natural join** takes **natural join** course;
- Wrong... only lists courses when the student took courses in their department (major)
- Correct:
  - **select** name, title  
**from** student **natural join** takes, course  
**where** takes.course\_id = course.course\_id;

[A. Silberschatz et al.]

# Outer Join

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- Joins so far are inner joins
- Outer joins returns tuples from one (or both) relations that do not match tuples in the other relation
- Fills in missing values with null
- Three forms of outer join:
  - **left** outer join
  - **right** outer join
  - full **outer** join

[A. Silberschatz et al.]



# Join Examples

<i>course_id</i>	<i>title</i>	<i>dept_name</i>	<i>credits</i>
BIO-301	Genetics	Biology	4
CS-190	Game Design	Comp. Sci.	4
CS-315	Robotics	Comp. Sci.	3

course

<i>course_id</i>	<i>prereq_id</i>
BIO-301	BIO-101
CS-190	CS-101
CS-347	CS-101

prereq

Left Join

<i>course_id</i>	<i>title</i>	<i>dept_name</i>	<i>credits</i>	<i>prereq_id</i>
BIO-301	Genetics	Biology	4	BIO-101
CS-190	Game Design	Comp. Sci.	4	CS-101
CS-315	Robotics	Comp. Sci.	3	<i>null</i>

Right Join

<i>course_id</i>	<i>title</i>	<i>dept_name</i>	<i>credits</i>	<i>prereq_id</i>
BIO-301	Genetics	Biology	4	BIO-101
CS-190	Game Design	Comp. Sci.	4	CS-101
CS-347	<i>null</i>	<i>null</i>	<i>null</i>	CS-101

[A. Silberschatz et al.]

# Join Examples

<i>course_id</i>	<i>title</i>	<i>dept_name</i>	<i>credits</i>
BIO-301	Genetics	Biology	4
CS-190	Game Design	Comp. Sci.	4
CS-315	Robotics	Comp. Sci.	3

course

<i>course_id</i>	<i>prereq_id</i>
BIO-301	BIO-101
CS-190	CS-101
CS-347	CS-101

prereq

(Full) Outer Join

<i>course_id</i>	<i>title</i>	<i>dept_name</i>	<i>credits</i>	<i>prereq_id</i>
BIO-301	Genetics	Biology	4	BIO-101
CS-190	Game Design	Comp. Sci.	4	CS-101
CS-315	Robotics	Comp. Sci.	3	<i>null</i>
CS-347	<i>null</i>	<i>null</i>	<i>null</i>	CS-101

Inner Join

<i>course_id</i>	<i>title</i>	<i>dept_name</i>	<i>credits</i>	<i>prereq_id</i>	<i>course_id</i>
BIO-301	Genetics	Biology	4	BIO-101	BIO-301
CS-190	Game Design	Comp. Sci.	4	CS-101	CS-190

[A. Silberschatz et al.]



# Arrays

---

What is the difference between an array and a list (or a tuple)?

# Arrays

---

- Usually a fixed size—lists are meant to change size
- Are mutable—tuples are not
- Store only one type of data—lists and tuples can store anything
- Are faster to access and manipulate than lists or tuples
- Can be multidimensional:
  - Can have list of lists or tuple of tuples but no guarantee on shape
  - Multidimensional arrays are rectangles, cubes, etc.

# Why NumPy?

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- Fast **vectorized** array operations for data munging and cleaning, subsetting and filtering, transformation, and any other kinds of computations
- Common array algorithms like sorting, unique, and set operations
- Efficient descriptive statistics and aggregating/summarizing data
- Data alignment and relational data manipulations for merging and joining together heterogeneous data sets
- Expressing conditional logic as array expressions instead of loops with `if-elif-else` branches
- Group-wise data manipulations (aggregation, transformation, function application).

[W. McKinney, Python for Data Analysis]

```
import numpy as np
```

# PyData Notebooks

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- <https://github.com/wesm/pydata-book/>
- ch04.ipynb
- Click the raw button and save that file to disk
- ...or download/clone the entire repository

# Creating arrays

---

- `data1 = [6, 7, 8, 0, 1]`  
`arr1 = np.array(data1)`
- `data2 = [[1.5, 2, 3, 4], [5, 6, 7, 8]]`  
`arr2 = np.array(data2)`
- `data3 = np.array([6, "abc", 3.57])` # !!! check !!!
- Can check the type of an array in `dtype` property
- Types:
  - `arr1.dtype` # `dtype('int64')`
  - `arr3.dtype` # `dtype('<U21')`, unicode plus # chars



# Types

---

- "But I thought Python wasn't stingy about types..."
- numpy aims for speed
- Able to do array arithmetic
- int16, int32, int64, float32, float64, bool, object
- Can specify type explicitly
  - `arr1_float = np.array(data1, dtype='float64')`
- `astype` method allows you to convert between different types of arrays:

```
arr = np.array([1, 2, 3, 4, 5])
arr.dtype
float_arr = arr.astype(np.float64)
```

# numpy data types (dtypes)

Type	Type code	Description
int8, uint8	i1, u1	Signed and unsigned 8-bit (1 byte) integer types
int16, uint16	i2, u2	Signed and unsigned 16-bit integer types
int32, uint32	i4, u4	Signed and unsigned 32-bit integer types
int64, uint64	i8, u8	Signed and unsigned 64-bit integer types
float16	f2	Half-precision floating point
float32	f4 or f	Standard single-precision floating point; compatible with C float
float64	f8 or d	Standard double-precision floating point; compatible with C double and Python float object
float128	f16 or g	Extended-precision floating point
complex64, complex128, complex256	c8, c16, c32	Complex numbers represented by two 32, 64, or 128 floats, respectively
bool	?	Boolean type storing True and False values
object	O	Python object type; a value can be any Python object
string_	S	Fixed-length ASCII string type (1 byte per character); for example, to create a string dtype with length 10, use 'S10'
unicode_	U	Fixed-length Unicode type (number of bytes platform specific); same specification semantics as string_ (e.g., 'U10')

[W. McKinney, Python for Data Analysis]



# Speed Benefits

---

- Compare random number generation in pure Python versus numpy

- Python:

- `import random`  
`%timeit rolls_list = [random.randrange(1,7)`  
`for i in range(0, 60_000)]`

- With NumPy:

- `%timeit rolls_array = np.random.randint(1, 7, 60_000)`

- Significant speedup (80x+)

# Array Shape

---

- Our normal way of checking the size of a collection is... `len`
- How does this work for arrays?
- `arr1 = np.array([1, 2, 3, 6, 9])`  
`len(arr1) # 5`
- `arr2 = np.array([[1.5, 2, 3, 4], [5, 6, 7, 8]])`  
`len(arr2) # 2`
- All dimension lengths → shape: `arr2.shape # (2, 4)`
- Number of dimensions: `arr2.ndim # 2`
- Can also reshape an array:
  - `arr2.reshape(4, 2)`
  - `arr2.reshape(-1, 2) # what happens here?`

# Array Programming

---

- Lists:

- ```
c = []  
  for i in range(len(a)):  
      c.append(a[i] + b[i])
```

- How to improve this?

# Array Programming

---

- Lists:

- `c = []`  
    `for i in range(len(a)):`  
        `c.append(a[i] + b[i])`
- `c = [aa + bb for aa, bb in zip(a,b)]`

- NumPy arrays:

- `c = a + b`

- More functional-style than imperative

- **Internal iteration** instead of external



# Operations

---

- `a = np.array([1, 2, 3])`  
`b = np.array([6, 4, 3])`
- (Array, Array) Operations (**Element-wise**)
  - Addition, Subtraction, Multiplication
  - `a + b` # `array([7, 6, 6])`
- (Scalar, Array) Operations (**Broadcasting**):
  - Addition, Subtraction, Multiplication, Division, Exponentiation
  - `a ** 2` # `array([1, 4, 9])`
  - `b + 3` # `array([9, 7, 6])`

# More on Array Creation

---

- Zeros: `np.zeros(10)`
- Ones: `np.ones((4,5))` # shape
- Empty: `np.empty((2,2))`
- \_like versions: pass an existing array and matches shape with specified contents
- Range: `np.arange(15)` # constructs an array, not iterator!

# Indexing

---

- Same as with lists plus shorthand for 2D+
  - `arr1 = np.array([6, 7, 8, 0, 1])`
  - `arr1[1]`
  - `arr1[-1]`
- What about two dimensions?
  - `arr2 = np.array([[1.5, 2, 3, 4], [5, 6, 7, 8]])`
  - `arr[1][1]`
  - `arr[1,1]` # shorthand

# 2D Indexing

|        |   | axis 1 |     |     |
|--------|---|--------|-----|-----|
|        |   | 0      | 1   | 2   |
| axis 0 | 0 | 0,0    | 0,1 | 0,2 |
|        | 1 | 1,0    | 1,1 | 1,2 |
|        | 2 | 2,0    | 2,1 | 2,2 |

[W. McKinney, Python for Data Analysis]

# Slicing

---

- 1D: Similar to lists
  - `arr1 = np.array([6, 7, 8, 0, 1])`
  - `arr1[2:5]` # `np.array([8, 0, 1])`, sort of
- Can **mutate** original array:
  - `arr1[2:5] = 3` # supports assignment
  - `arr1` # the original array changed
- Slicing returns **views** (copy the array if original array shouldn't change)
  - `arr1[2:5]` # a view
  - `arr1[2:5].copy()` # a new array

# Slicing

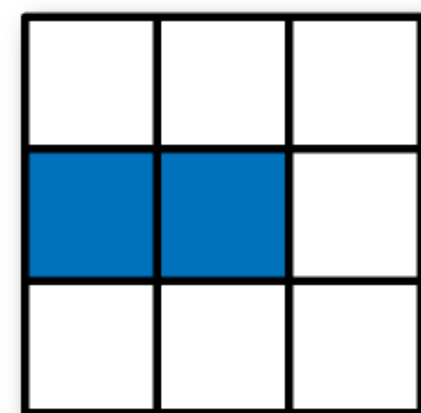
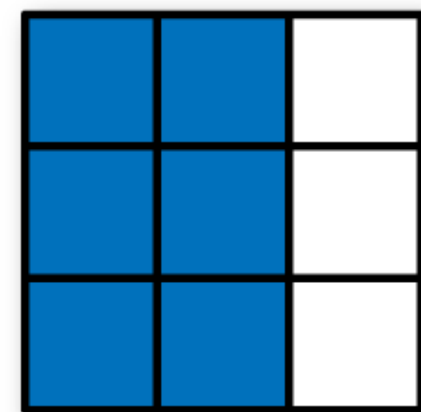
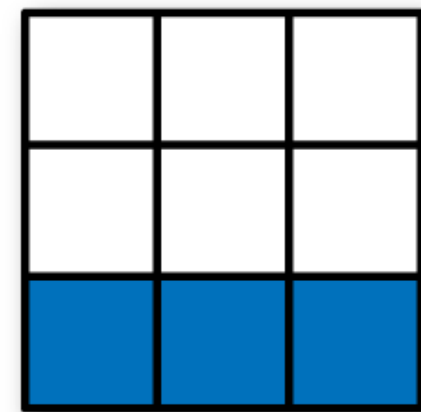
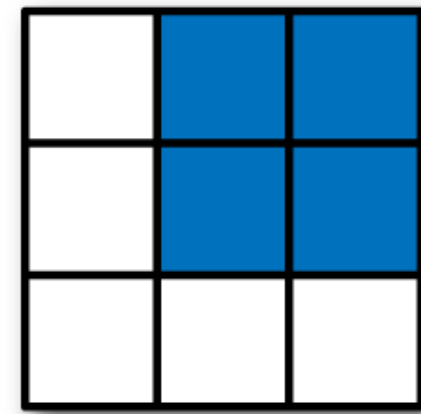
---

- 2D+: comma separated indices as shorthand:
  - `arr2 = np.array([[1.5, 2, 3, 4], [5, 6, 7, 8]])`
  - `a[1:3, 1:3]`
  - `a[1:3, :]` # works like in single-dimensional lists
- Can combine index and slice in different dimensions
  - `a[1, :]` # gives a row
  - `a[:, 1]` # gives a column



# 2D Array Slicing

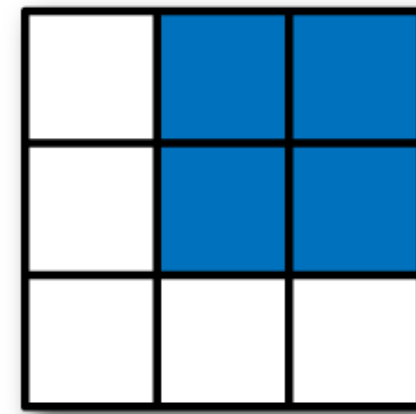
How to obtain the blue slice  
from array `arr`?



[W. McKinney, Python for Data Analysis]

# 2D Array Slicing

How to obtain the blue slice  
from array `arr`?

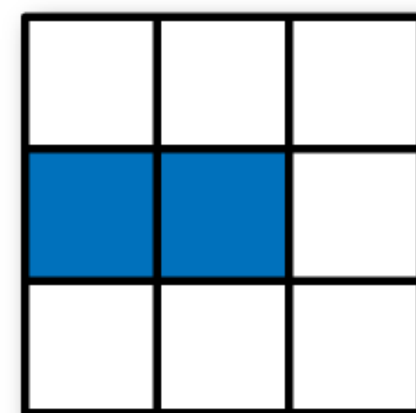
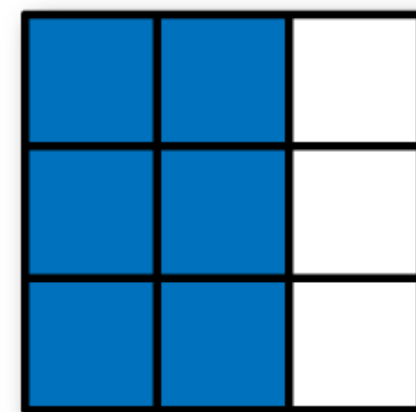
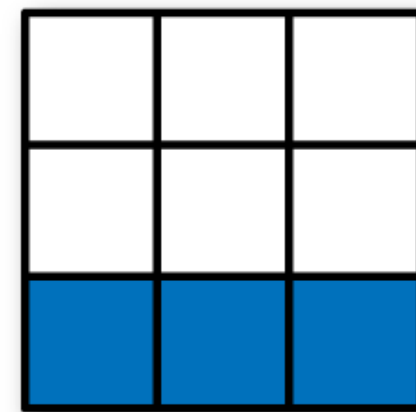


Expression

`arr[:2, 1:]`

Shape

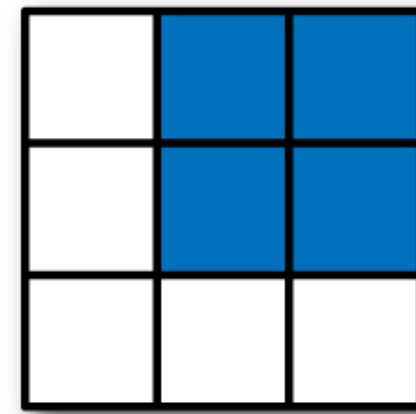
`(2, 2)`



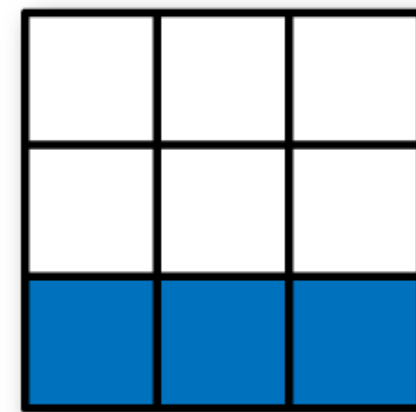
[W. McKinney, Python for Data Analysis]

# 2D Array Slicing

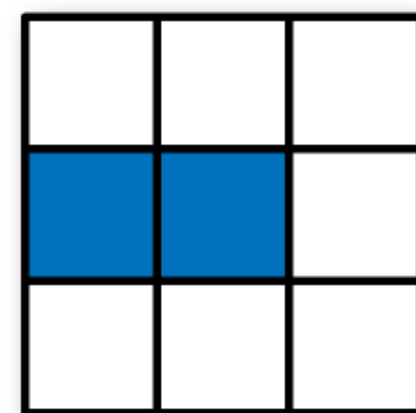
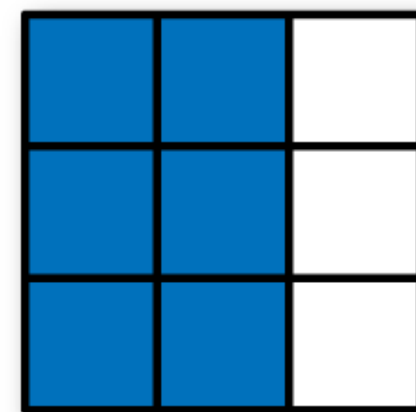
How to obtain the blue slice  
from array `arr`?



| Expression               | Shape               |
|--------------------------|---------------------|
| <code>arr[:2, 1:]</code> | <code>(2, 2)</code> |



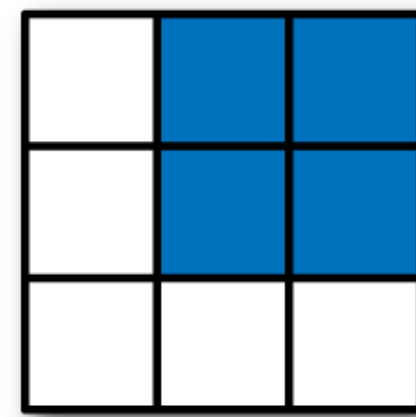
|                         |                     |
|-------------------------|---------------------|
| <code>arr[2]</code>     | <code>(3,)</code>   |
| <code>arr[2, :]</code>  | <code>(3,)</code>   |
| <code>arr[2:, :]</code> | <code>(1, 3)</code> |



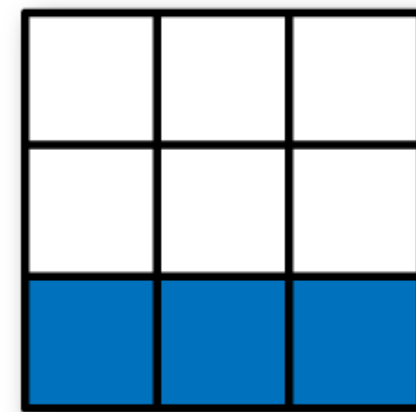
[W. McKinney, Python for Data Analysis]

# 2D Array Slicing

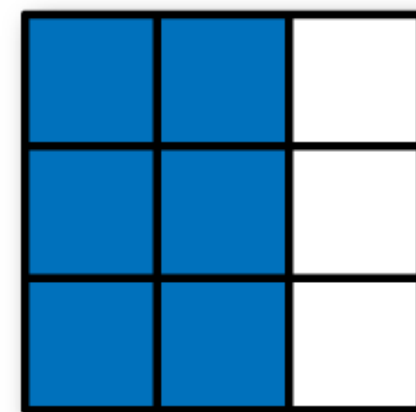
How to obtain the blue slice  
from array `arr`?



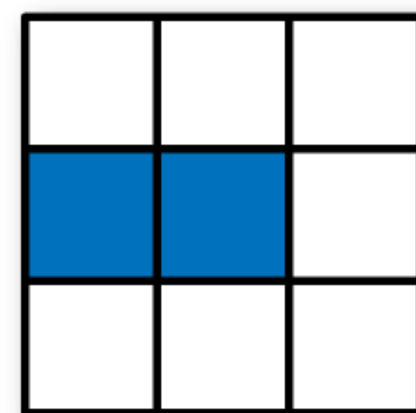
| Expression               | Shape               |
|--------------------------|---------------------|
| <code>arr[:2, 1:]</code> | <code>(2, 2)</code> |



|                         |                     |
|-------------------------|---------------------|
| <code>arr[2]</code>     | <code>(3,)</code>   |
| <code>arr[2, :]</code>  | <code>(3,)</code>   |
| <code>arr[2:, :]</code> | <code>(1, 3)</code> |



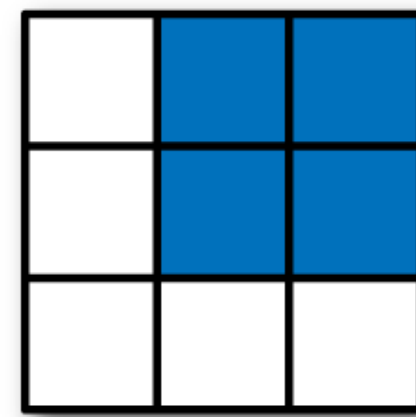
|                         |                     |
|-------------------------|---------------------|
| <code>arr[:, :2]</code> | <code>(3, 2)</code> |
|-------------------------|---------------------|



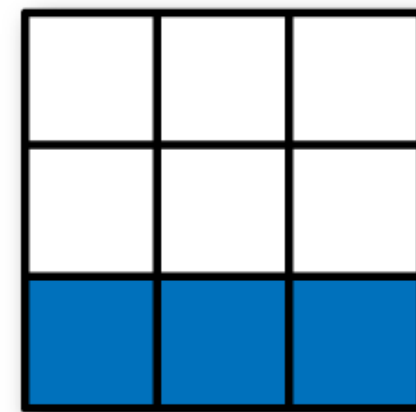
[W. McKinney, Python for Data Analysis]

# 2D Array Slicing

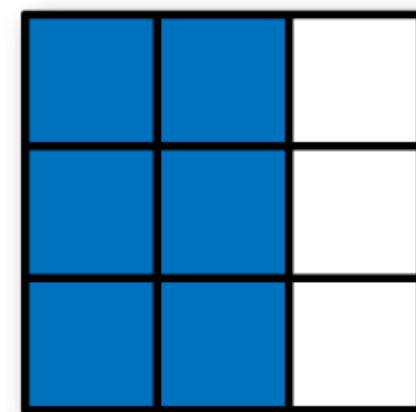
How to obtain the blue slice from array `arr`?



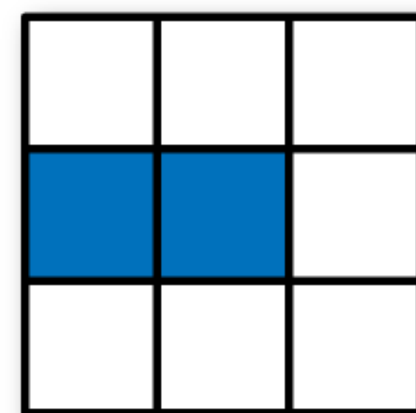
| Expression               | Shape               |
|--------------------------|---------------------|
| <code>arr[:2, 1:]</code> | <code>(2, 2)</code> |



|                         |                     |
|-------------------------|---------------------|
| <code>arr[2]</code>     | <code>(3,)</code>   |
| <code>arr[2, :]</code>  | <code>(3,)</code>   |
| <code>arr[2:, :]</code> | <code>(1, 3)</code> |



|                         |                     |
|-------------------------|---------------------|
| <code>arr[:, :2]</code> | <code>(3, 2)</code> |
|-------------------------|---------------------|



|                           |                     |
|---------------------------|---------------------|
| <code>arr[1, :2]</code>   | <code>(2,)</code>   |
| <code>arr[1:2, :2]</code> | <code>(1, 2)</code> |

[W. McKinney, Python for Data Analysis]