Data Visualization (CSCI 627/490)

Vector Field Visualization

Dr. David Koop

Volume Rendering vs. Isosurfacing

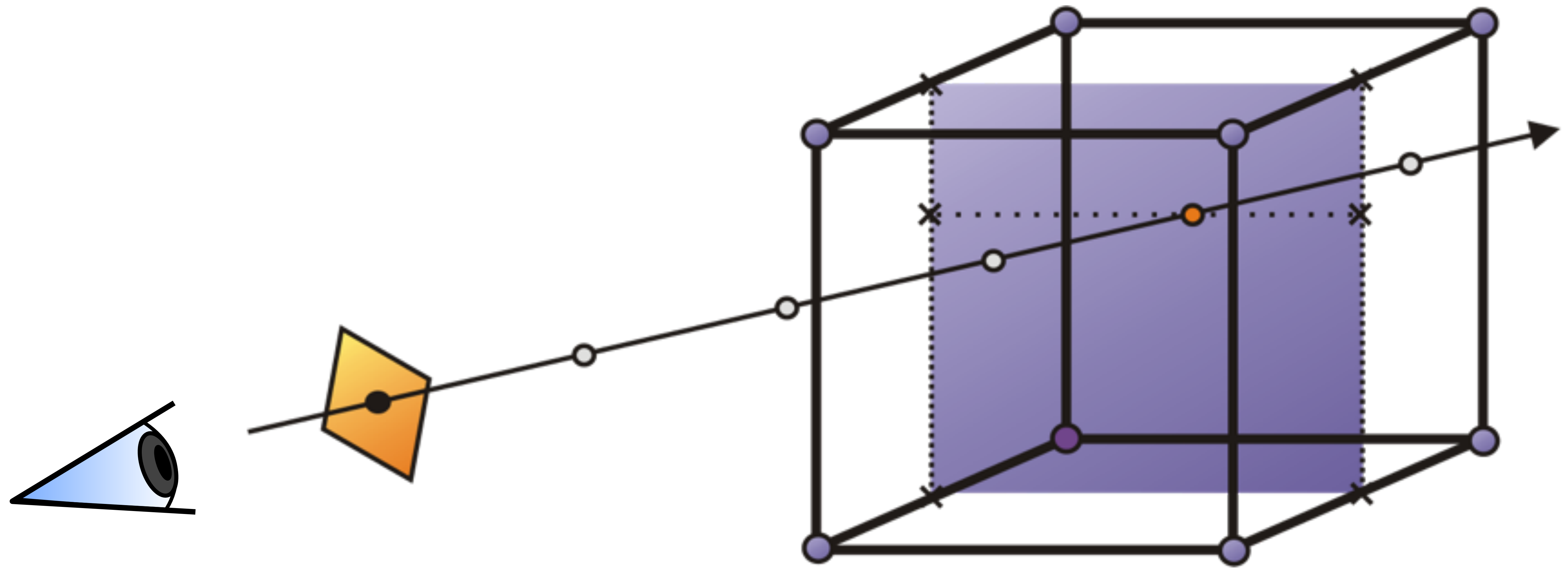
(a) Direct volume rendered

(b) Isosurface rendered

[Kindlmann, 1998]

How? Volume Ray Casting

- Approximate volume rendering integral: light absorption & emission
- Sample at regular intervals along each ray
- Trilinear interpolation: linear interpolation along each axes (x,y,z)

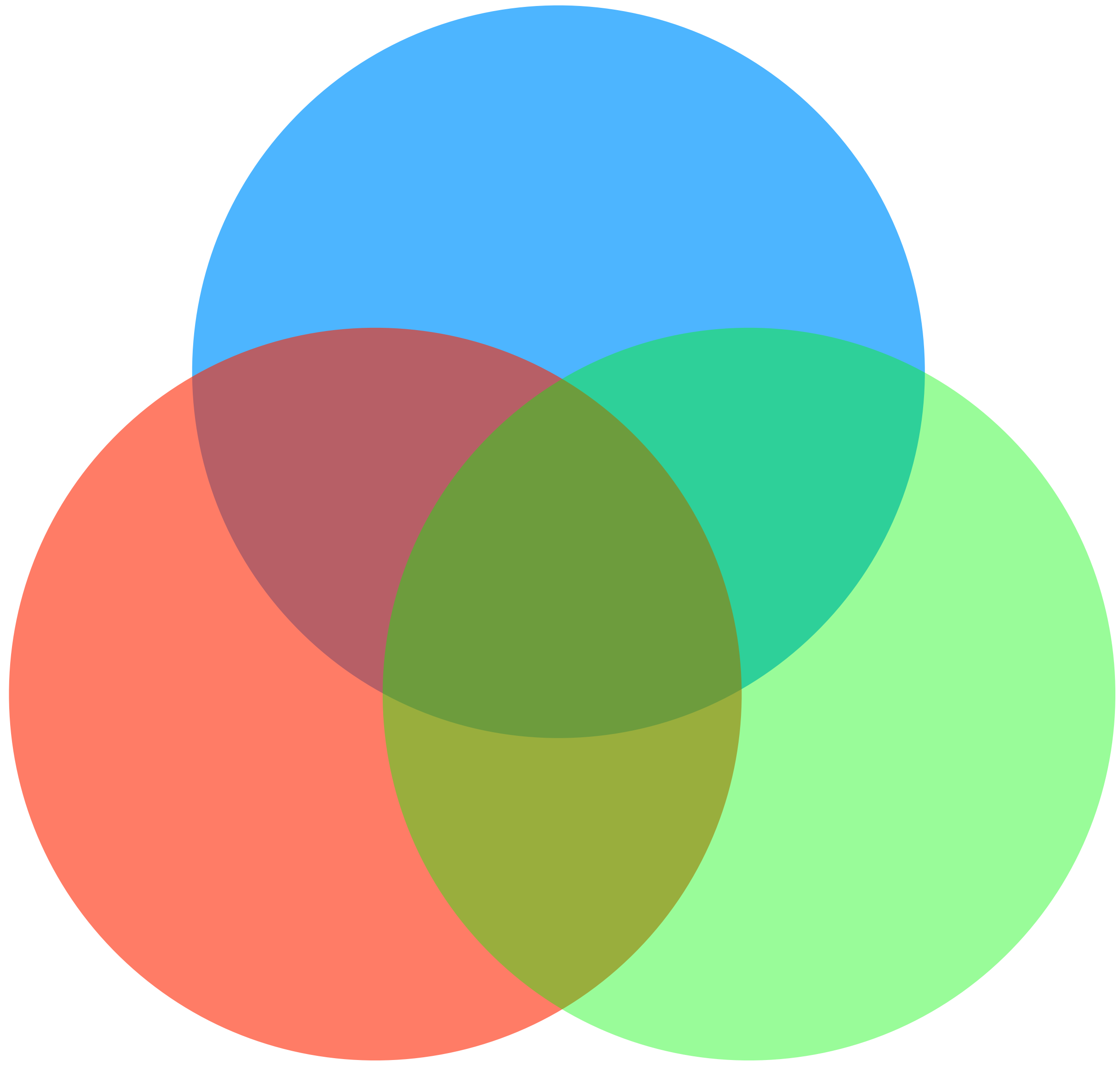


- Not the only possibility, also "object order" techniques like splatting or texture-based and combinations like shear-warp

Accumulation

- If we're not just calculating a single number (max, average) or a position (first), how do we determine the accumulation?
- Assume each value has an associated color (c) and opacity (α)
- Over operator (back-to-front):
  - $c = \alpha_f \cdot c_f + (1 - \alpha_f) \cdot \alpha_b \cdot c_b$
  - $\alpha = \alpha_f + (1 - \alpha_f) \cdot \alpha_b$
- Order is important!

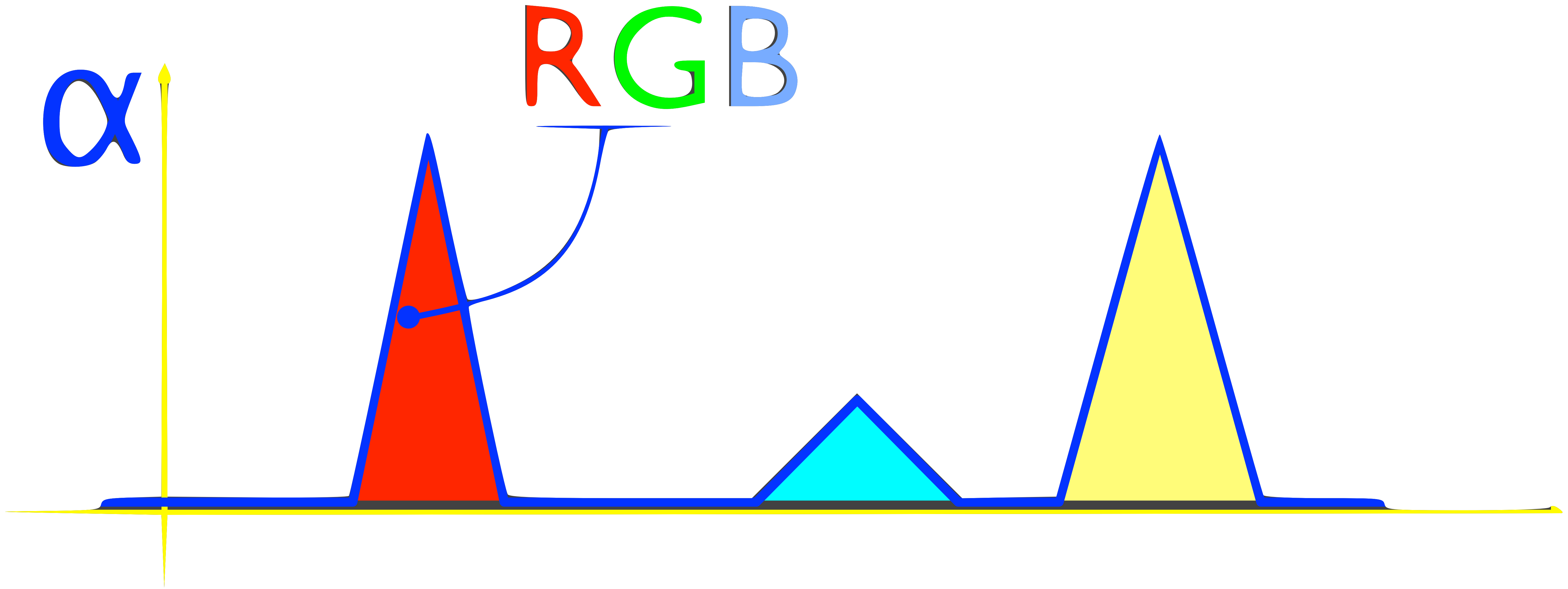
Blue Last



Blue First

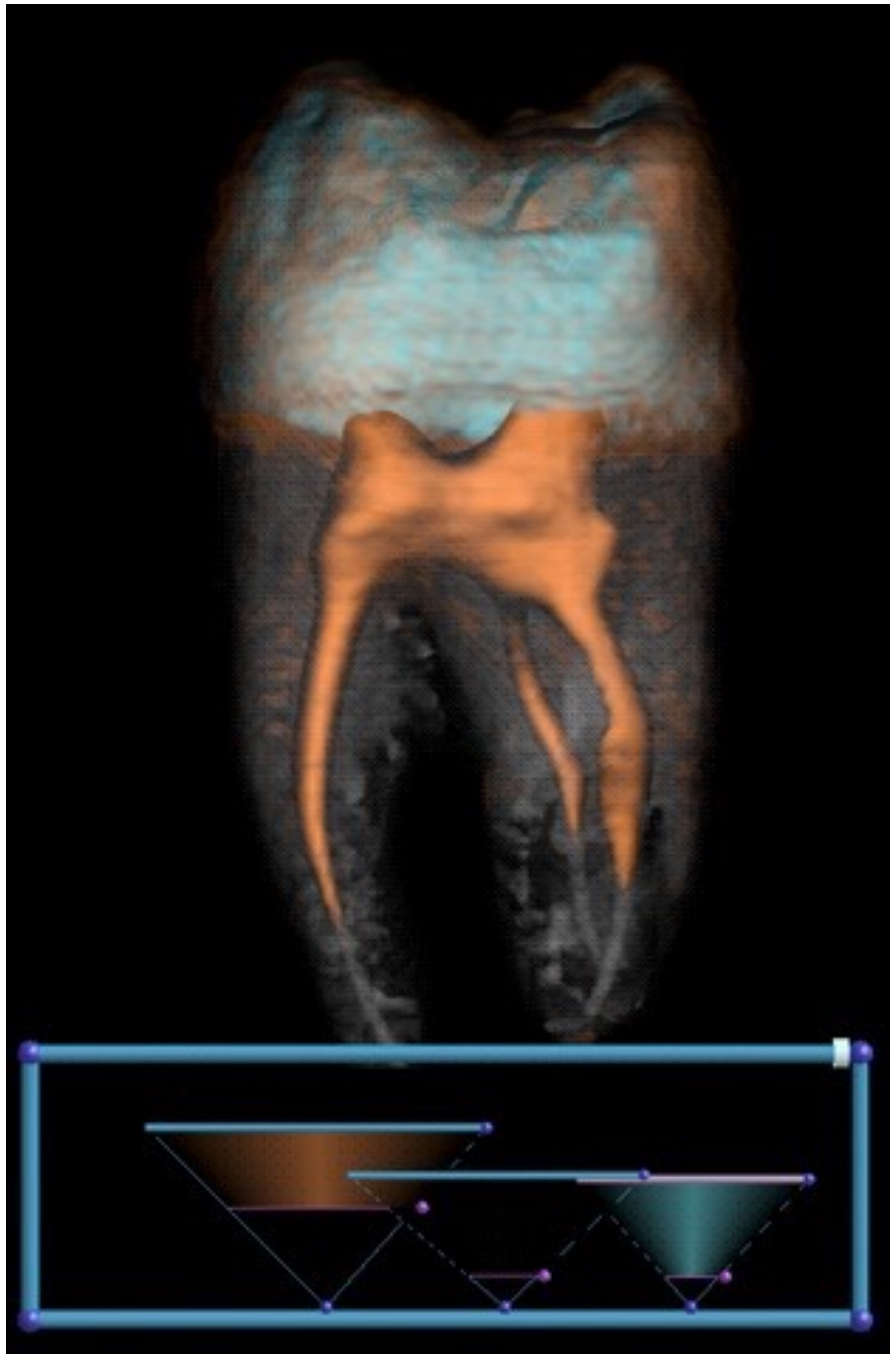
Transfer Functions

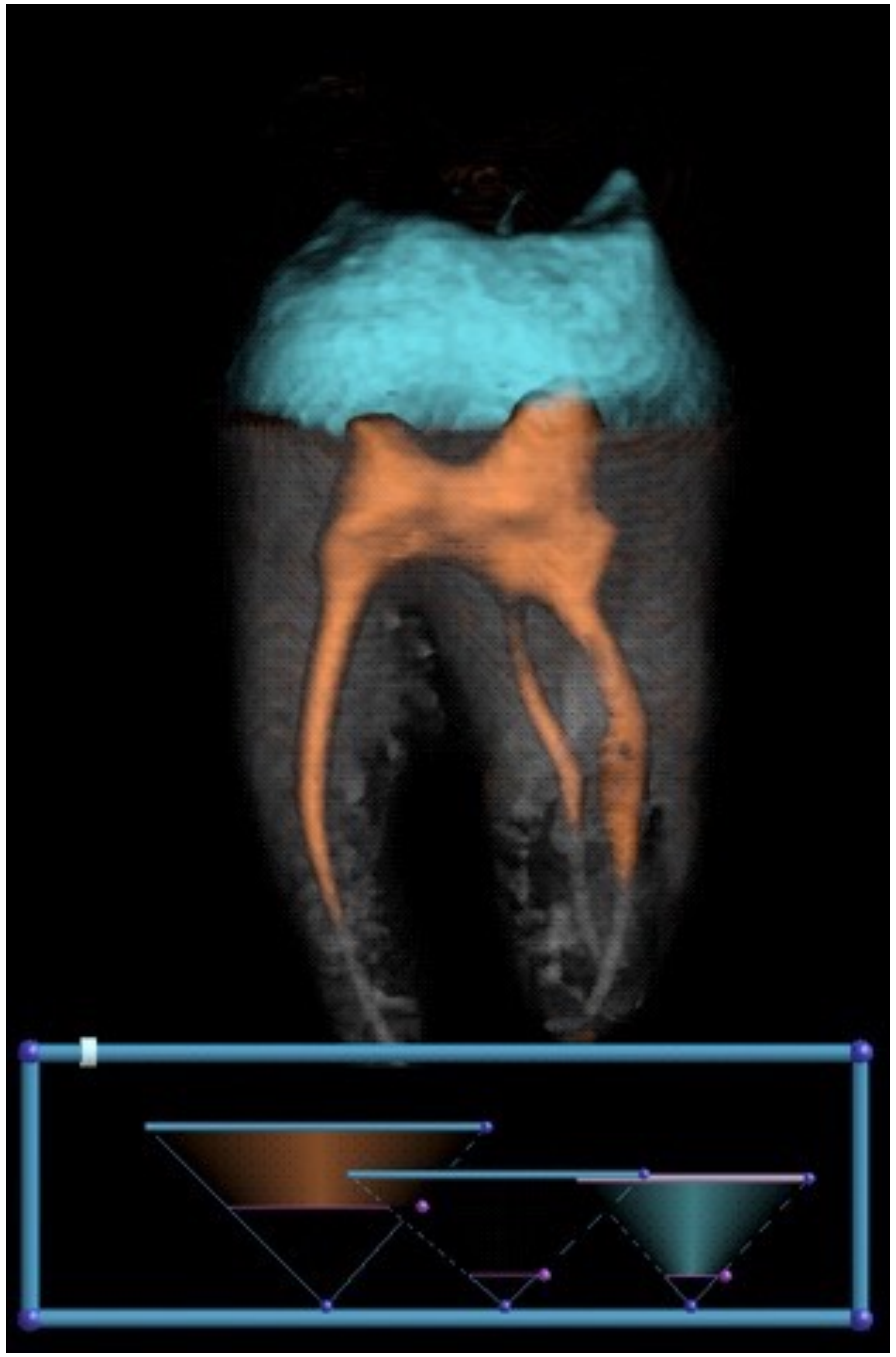
- Where do the colors and opacities come from?
- Idea is that each voxel emits/absorbs light based on its scalar value
- …but users get to choose how that happens
- x-axis: color region definitions, y-axis: opacity



[Kindlmann]

Multidimensional Transfer Functions

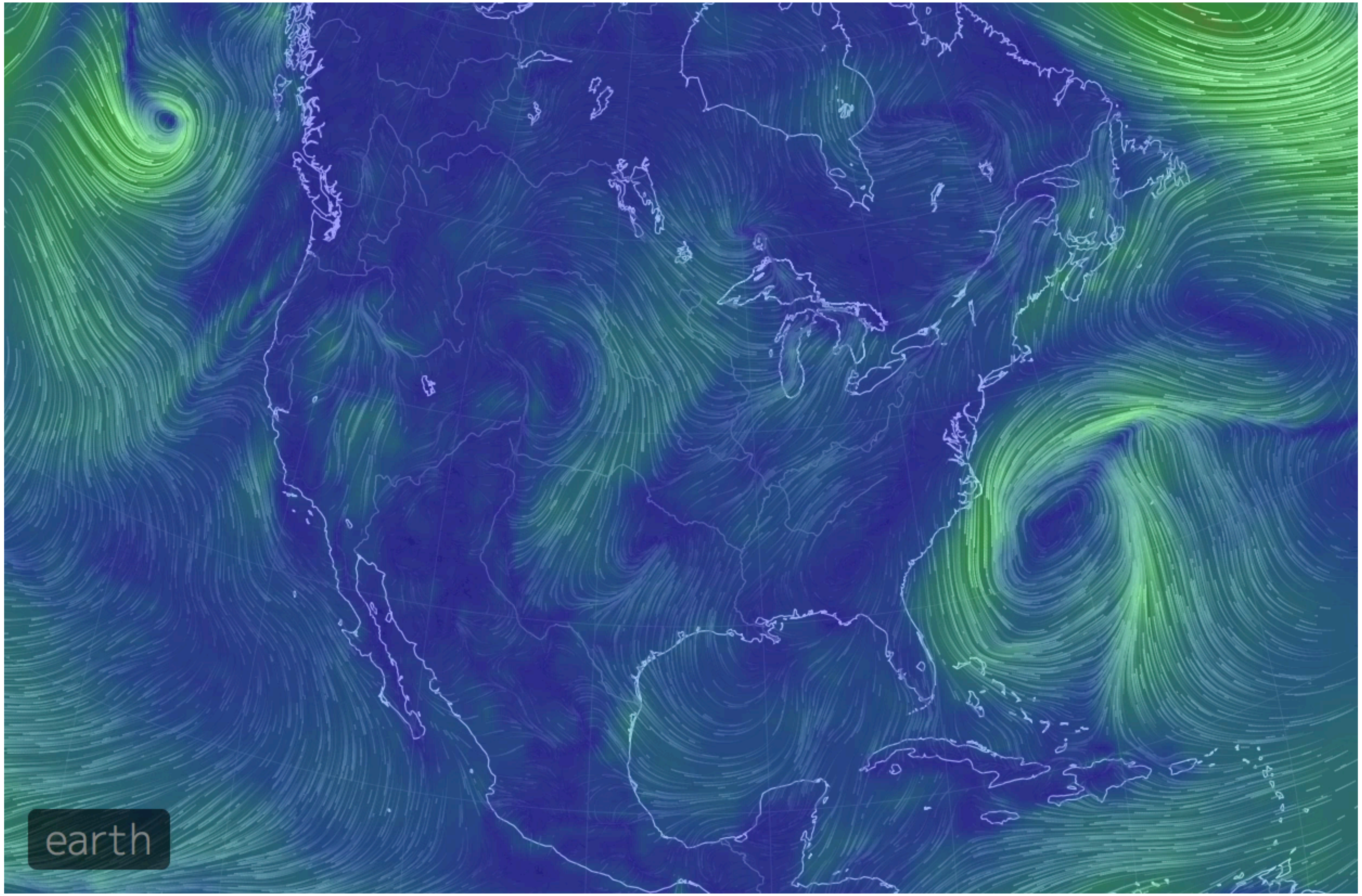




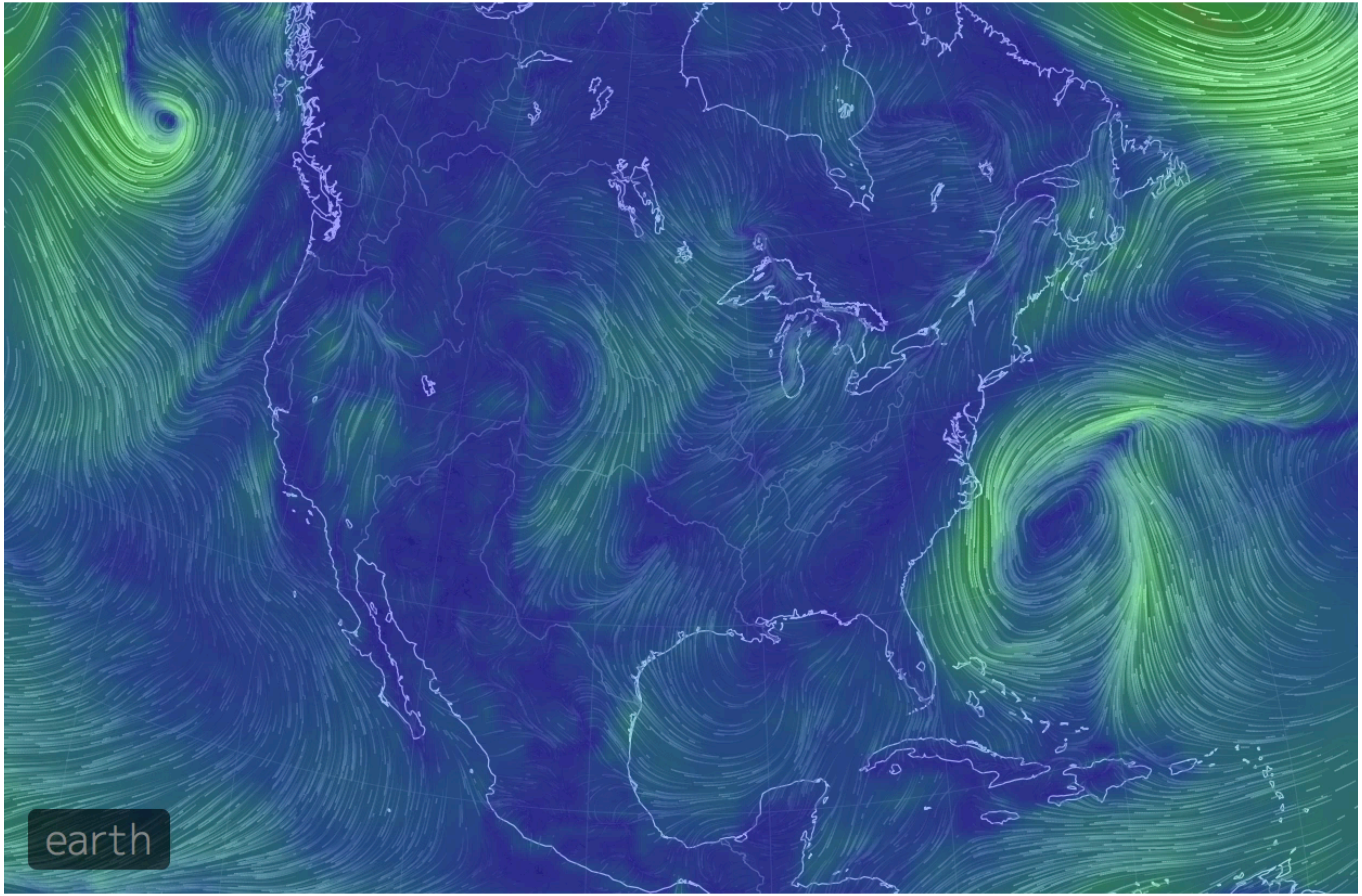
[J. Kniss]

Vector Field Visualization

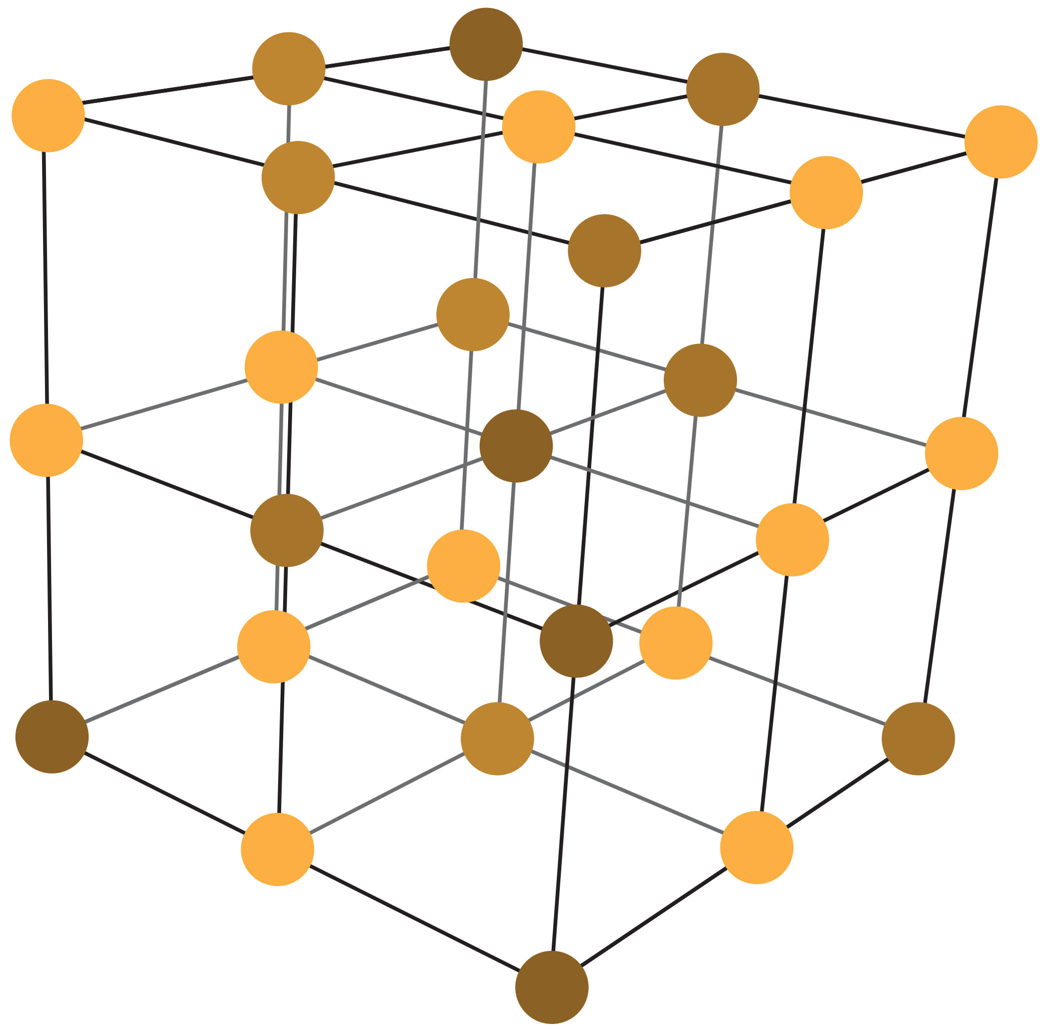
Examples of Vector Fields



Examples of Vector Fields

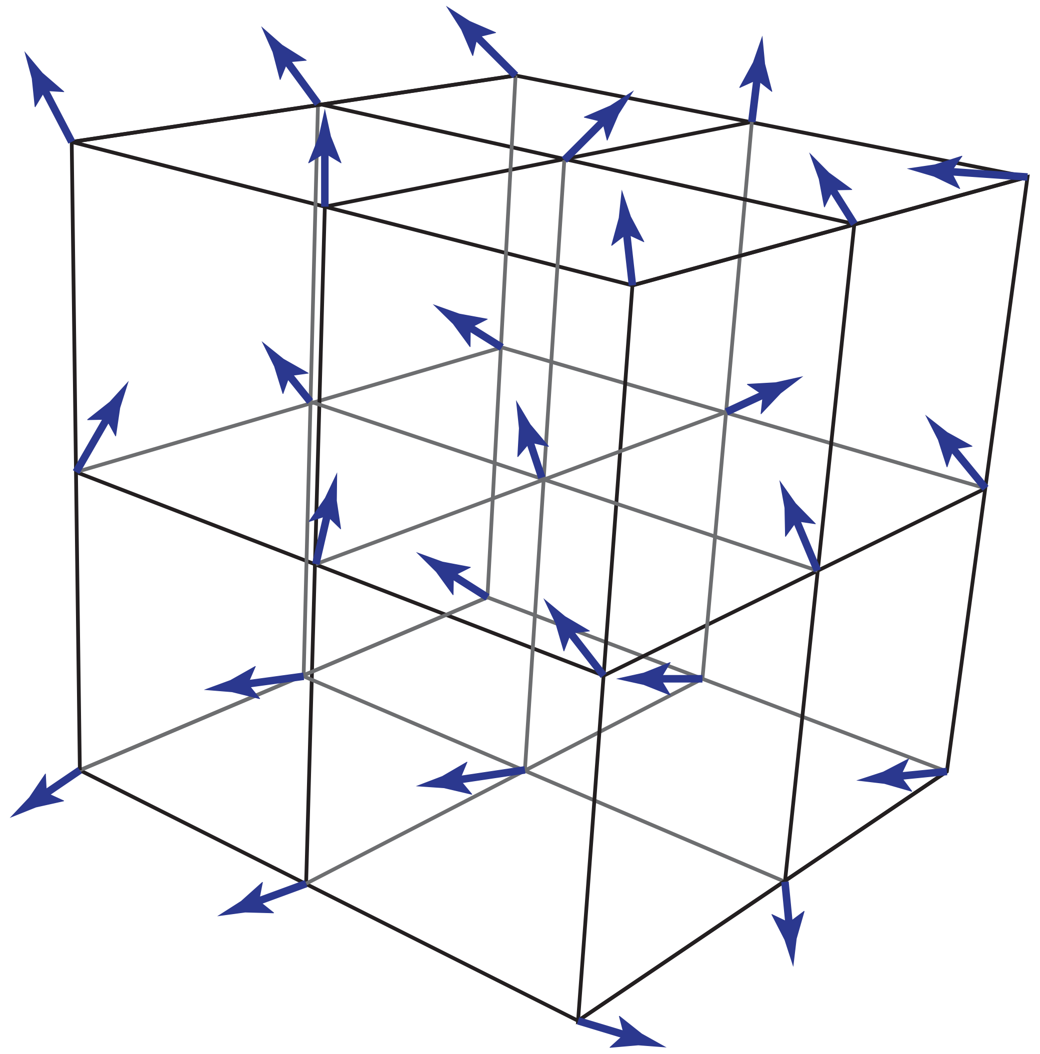


Fields in Visualization



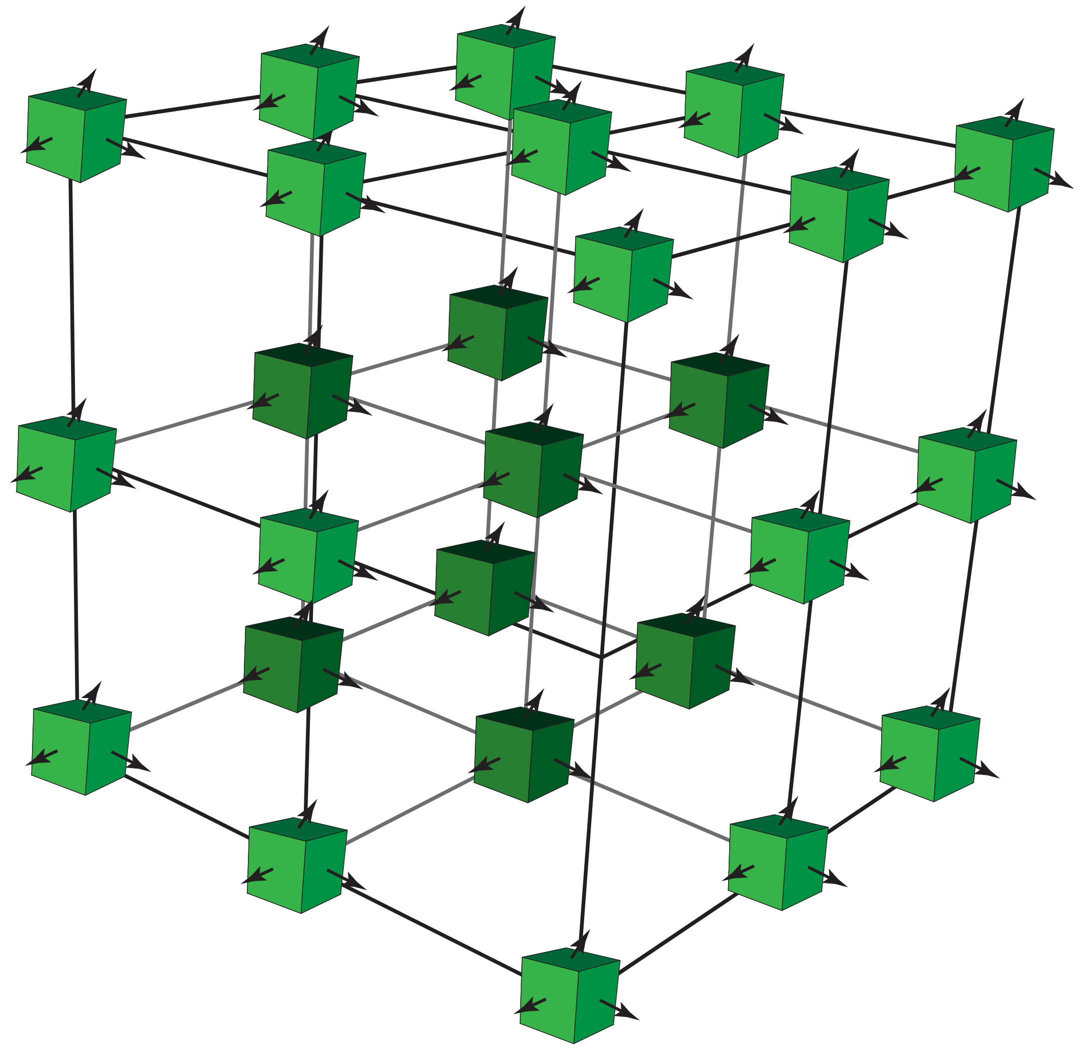
Scalar Fields

(Order-0 Tensor Fields)



Vector Fields

(Order-1 Tensor Fields)



Tensor Fields

(Order-2+)

Each point in space has an associated...

$$s_0$$

Scalar

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix}$$

Vector

$$\begin{bmatrix} \sigma_{00} & \sigma_{01} & \sigma_{02} \\ \sigma_{10} & \sigma_{11} & \sigma_{12} \\ \sigma_{20} & \sigma_{21} & \sigma_{22} \end{bmatrix}$$

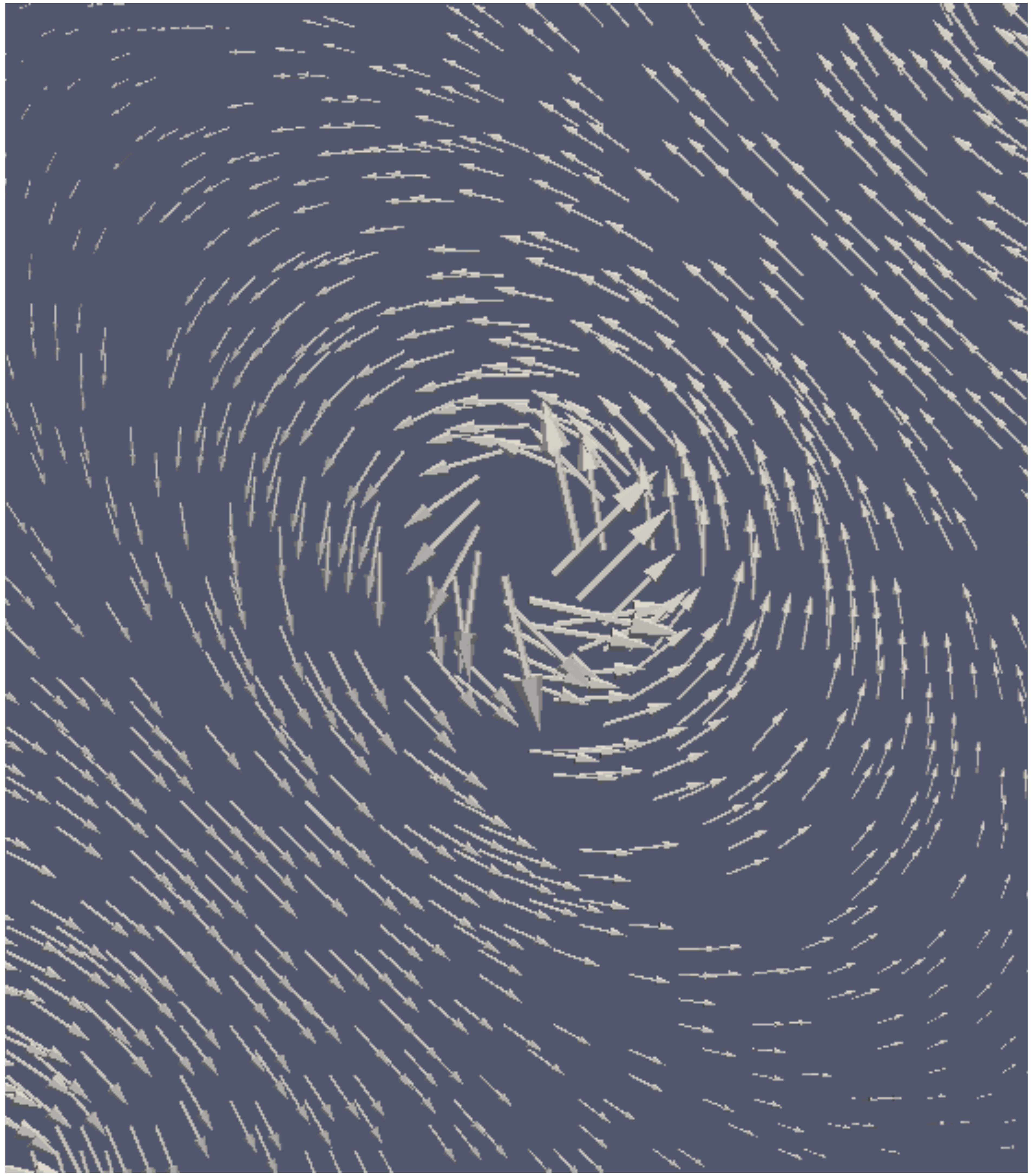
Tensor

Visualizing Vector Fields

- Direct: Glyphs, Render statistics as scalars
- Geometry: Streamlines and variants
- Textures: Line Integral Convolution (LIC)
- Topology: Extract relevant features and draw them

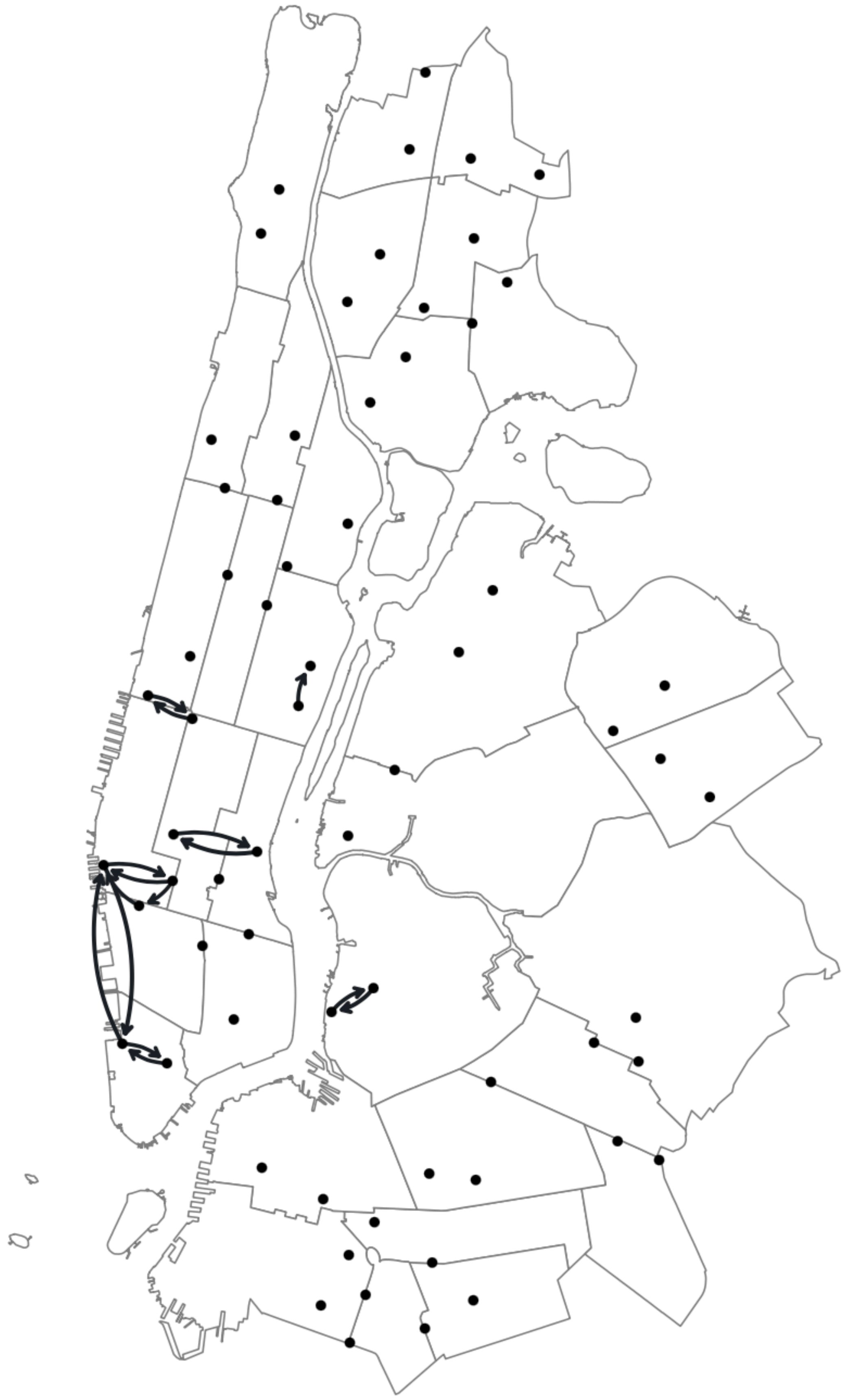
Glyphs

- Represent each vector with a symbol
- Hedgehogs are primitive glyphs (glyph is a line)
- Glyphs that show direction and/or magnitude can convey more information
- If we have a separate scalar value, how might we encode that?
- Clutter issues



Assignment 5

- Create Multiple Views
- Filtering
- Linked Highlighting
- Aggregation



Courselets

- Please provide feedback on the courselets if you have used them
- You can still work through them and complete them
- Extra credit for each completed survey

Final Project

- Designs feedback on Blackboard
- Work on implementations
- Presentations will be next week (April 28 and April 30)
- Submit information to Blackboard later this week
  - Project or link to project
  - Preference for Monday or Wednesday presentation
- Reports due at the end of the class

Final Exam

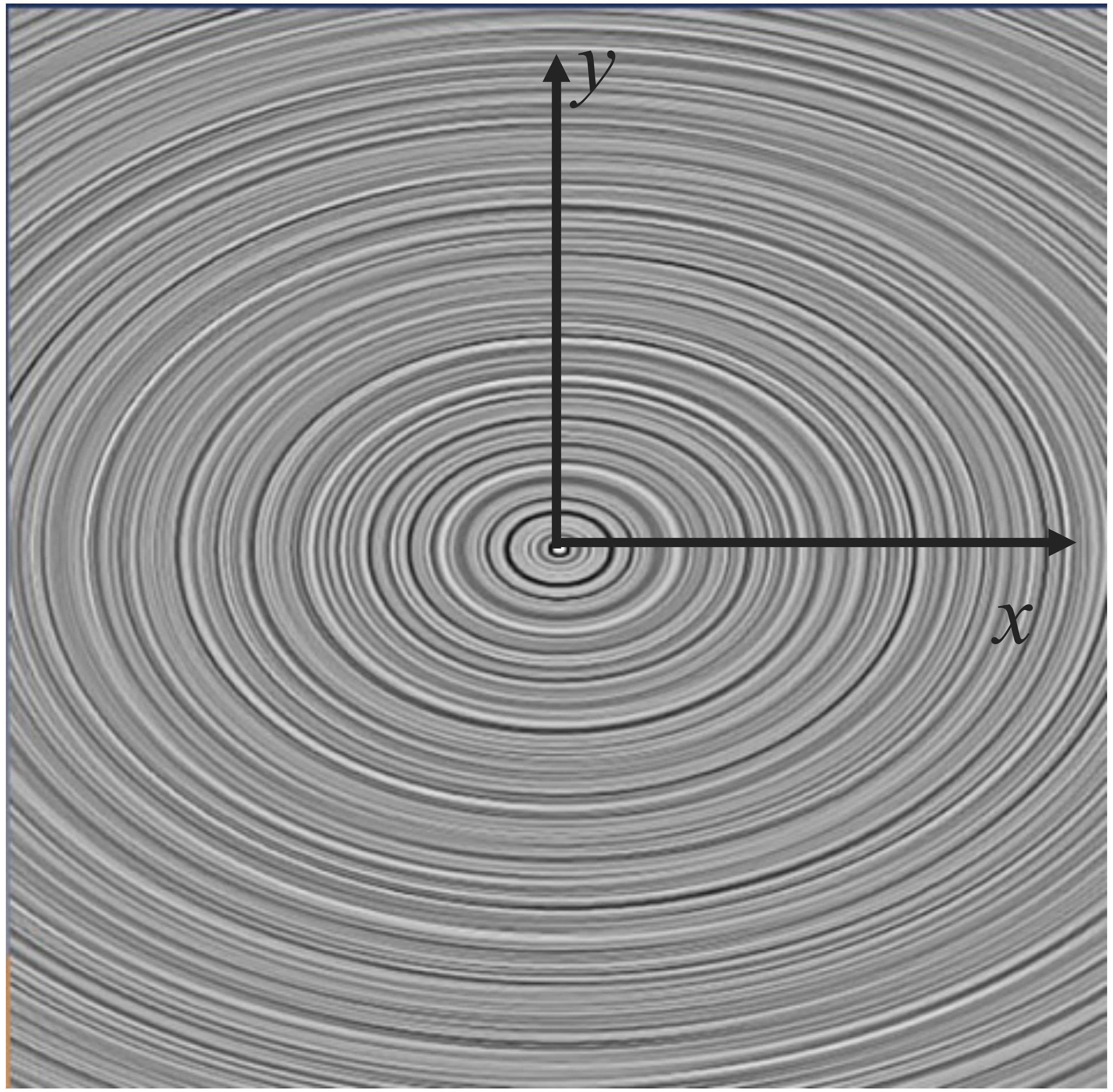
- Wednesday, May 7, 2025, **8:00-9:50pm**
- Covers all topics but emphasizes second half of the course
- Similar format as Midterm (multiple choice, free response)
- 627 Students will have a extra questions related to the research papers

Streamlines & Variants

- Trace a line along the direction of the vectors
- Streamlines are always tangent to the vector field
- Basic Particle Tracing:
  1. Set a starting point (seed)
  2. Take a step in the direction of the vector at that point
  3. Adjust direction based on the vector where you are now
  4. Go to Step 2 and Repeat

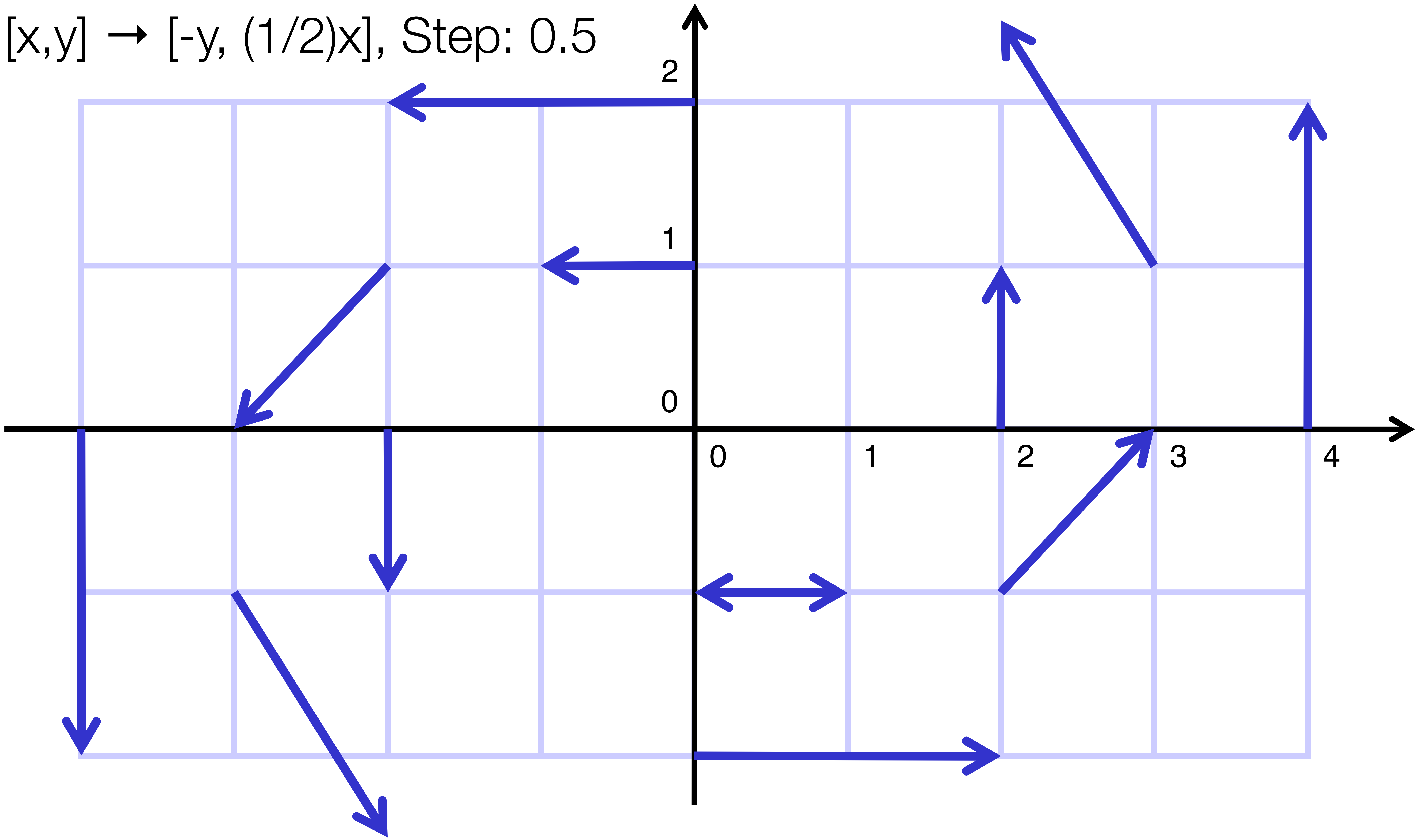
Example

- Elliptical path
- Suppose we have the actual equation
- Given point (x,y), the vector is at that point is $[v_x, v_y]$ where
  - $v_x = -y$
  - $v_y = (1/2)x$
- Want a streamline starting at (0,-1)



[LIC (not streamlines!) via Levine]

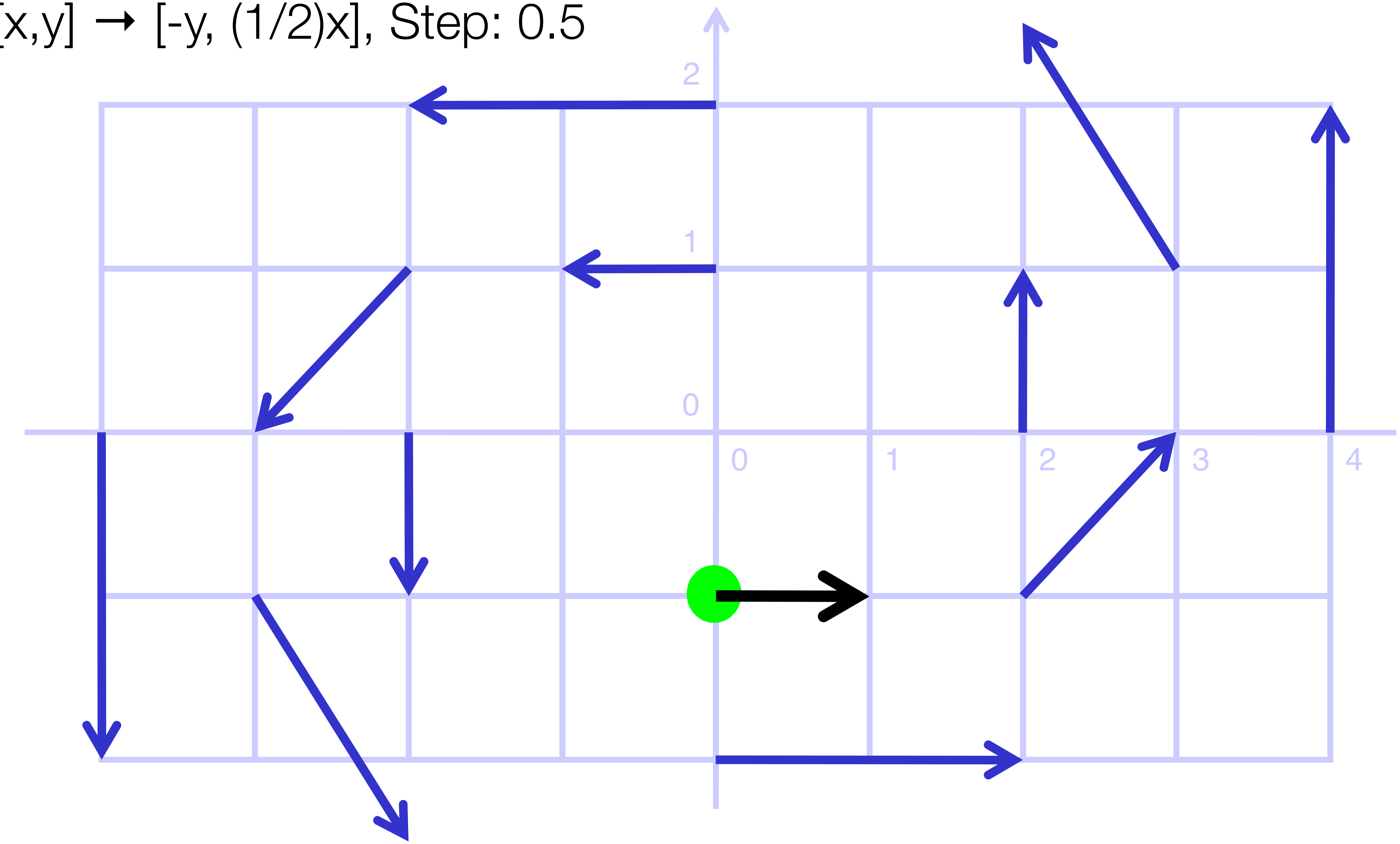
Some Glyphs



[via Levine]

Streamlines (Step 1)

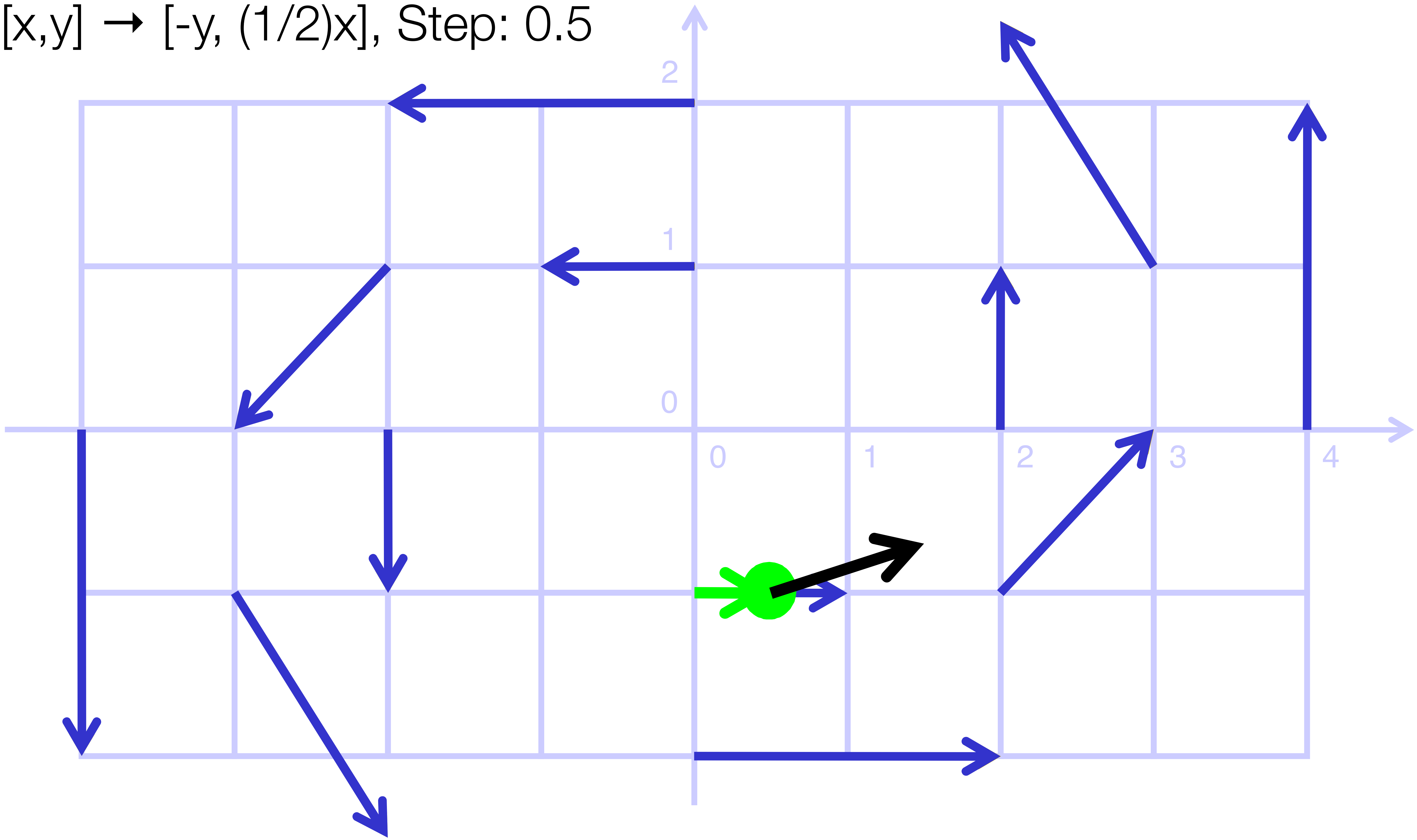
$[x,y] \rightarrow [-y, (1/2)x]$, Step: 0.5



[via Levine]

Streamlines (Step 2)

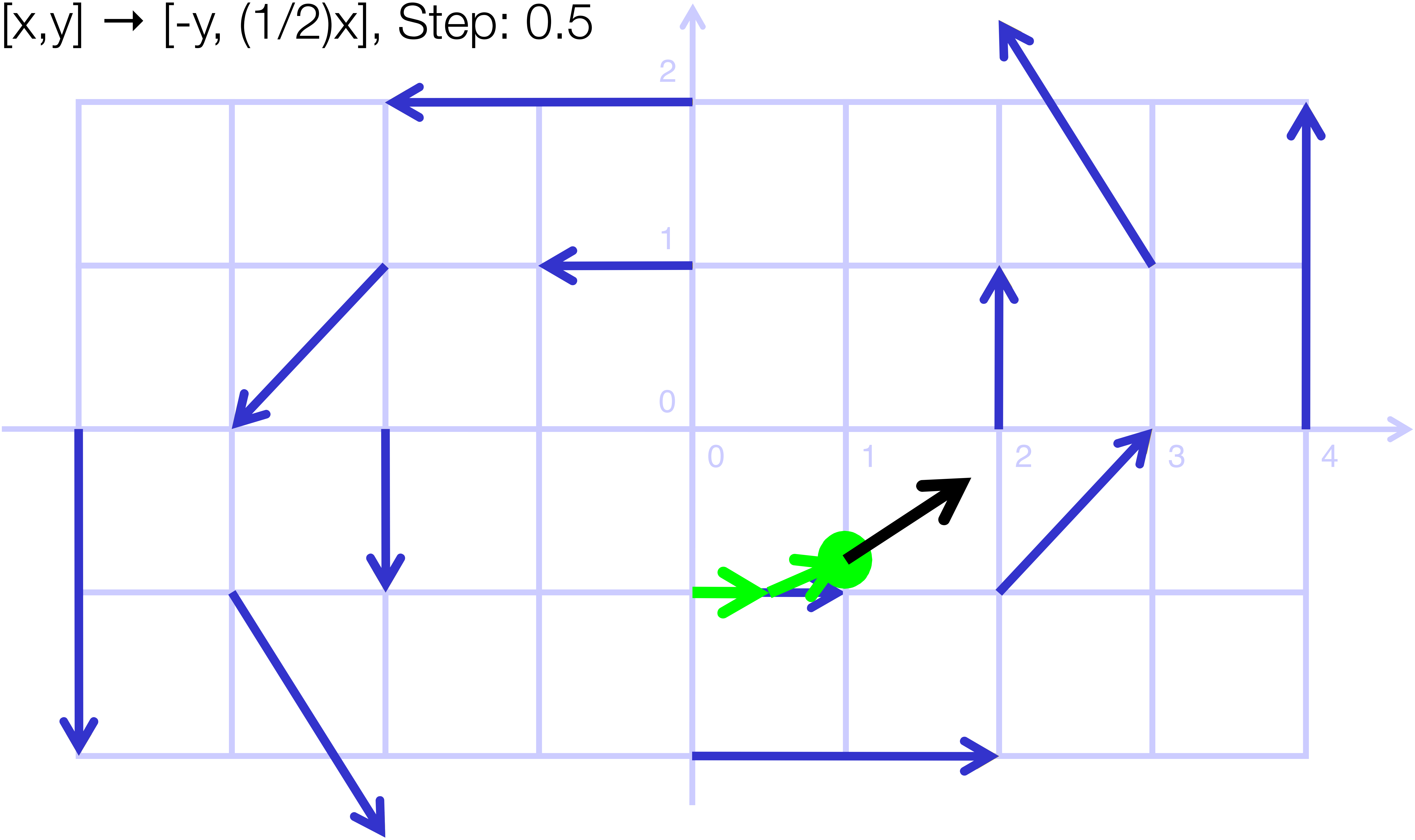
$[x,y] \rightarrow [-y, (1/2)x]$, Step: 0.5



[via Levine]

Streamlines (Step 3)

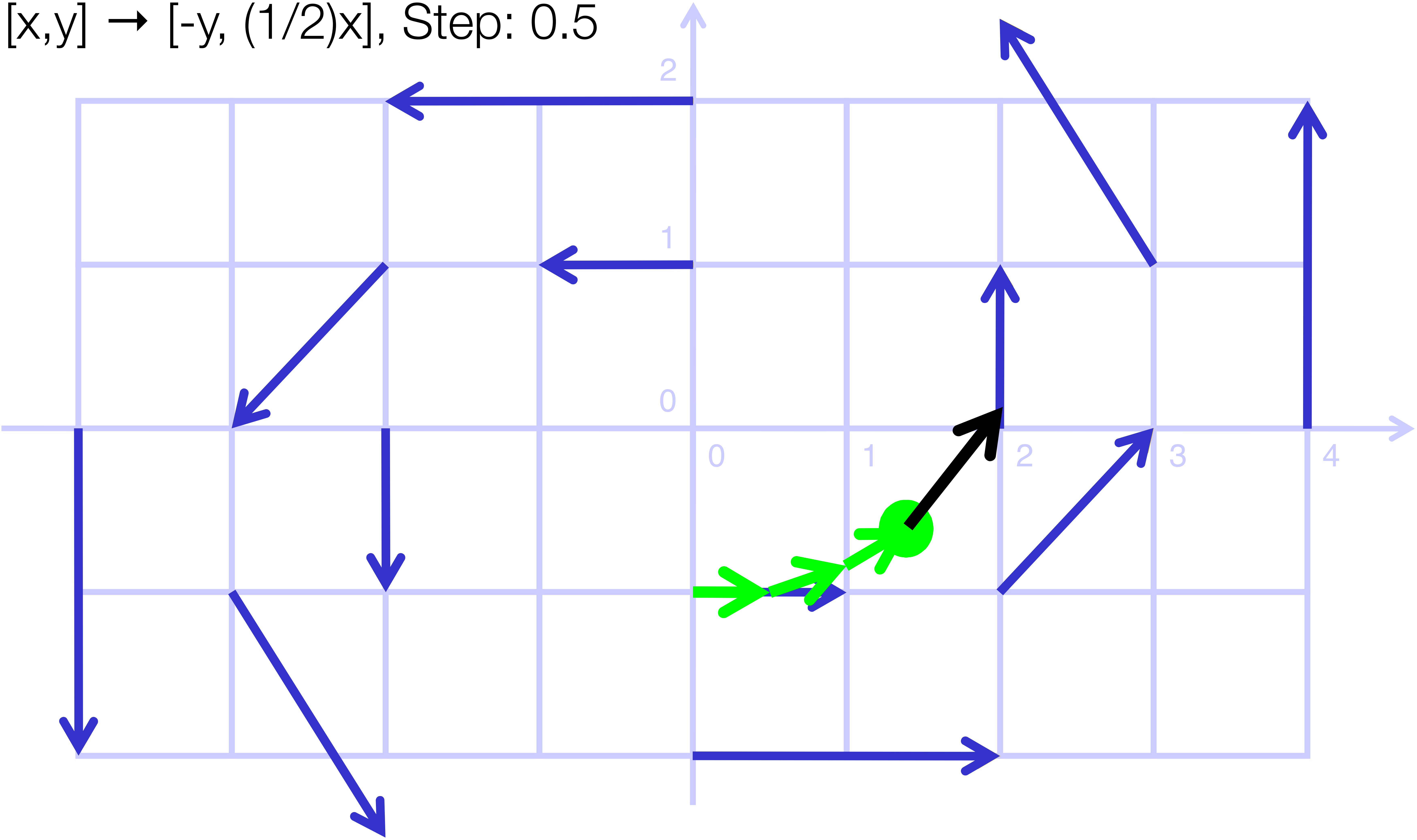
$[x,y] \rightarrow [-y, (1/2)x]$, Step: 0.5



[via Levine]

Streamlines (Step 4)

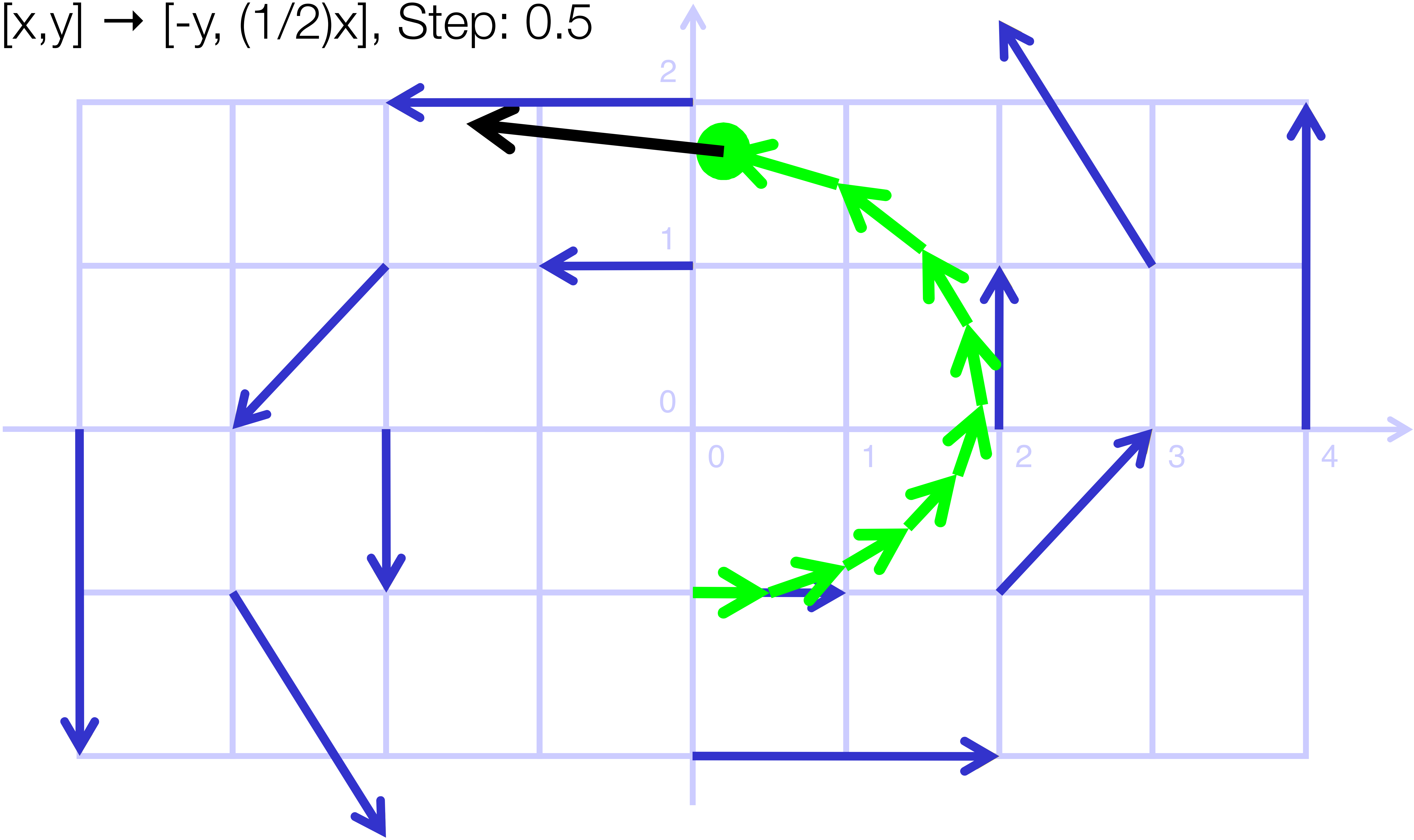
$[x,y] \to [-y, (1/2)x]$, Step: 0.5



[via Levine]

Streamlines (Step 10)

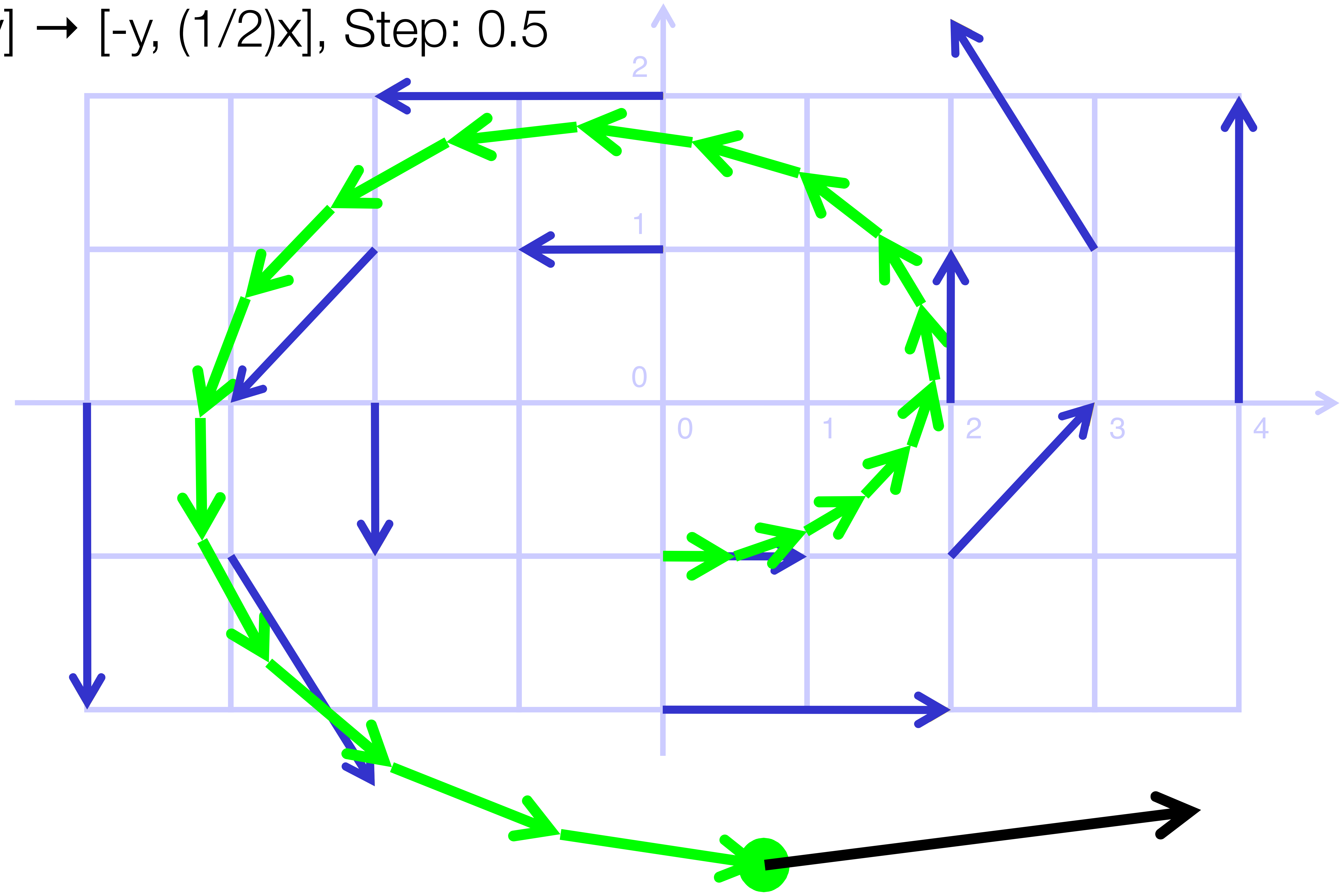
[x,y] → [-y, (1/2)x], Step: 0.5



[via Levine]

Streamlines (Step 19)

$[x,y] \rightarrow [-y, (1/2)x]$, Step: 0.5



[via Levine]

Euler Method

- Seeking to approximate integration of the velocity over time
- Euler method is the starting point for approximating this
- Problems?

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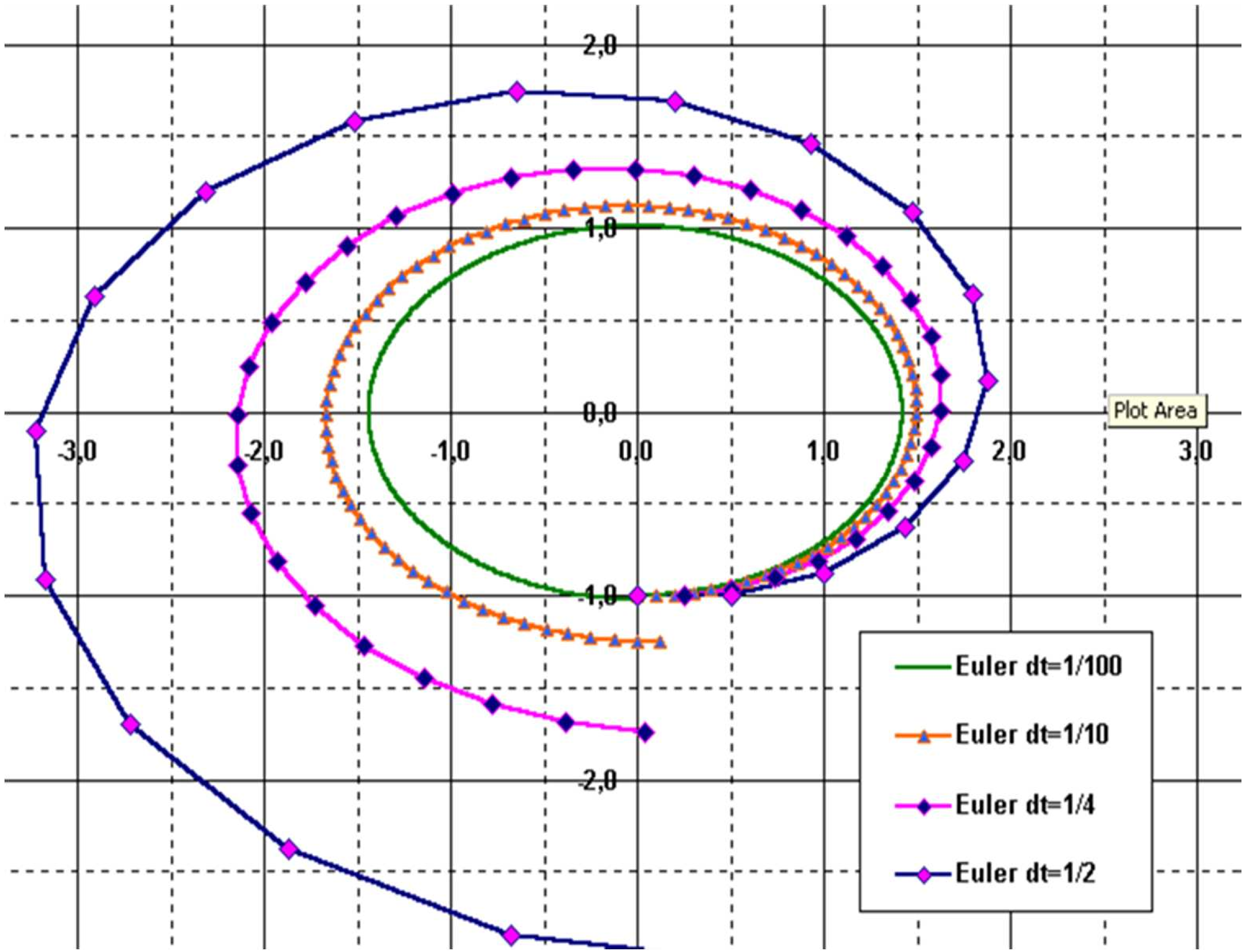
Euler Method

- Seeking to approximate integration of the velocity over time
- Euler method is the starting point for approximating this
- Problems?
  - Choice of step size is important
  - Choice of seed points are important
- Also remember that we have a field—we don't have measurements at every point (interpolation)

Euler Quality by Step Size



[via Levine]

Numerical Integration

- How do we generate accurate streamlines?
- Solving an ordinary differential equation

$$\frac{dL}{dt} = v(L(t)) \qquad L(0) = L_0$$

where L is the streamline, v is the vector field, and t is "time"

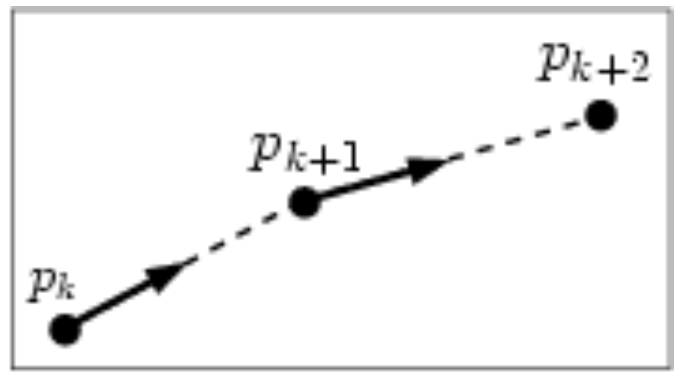
- Solution:

$$L(t + \Delta t) = L(t) + \boxed{\int_t^{t+\Delta t} v(L(t))dt}$$

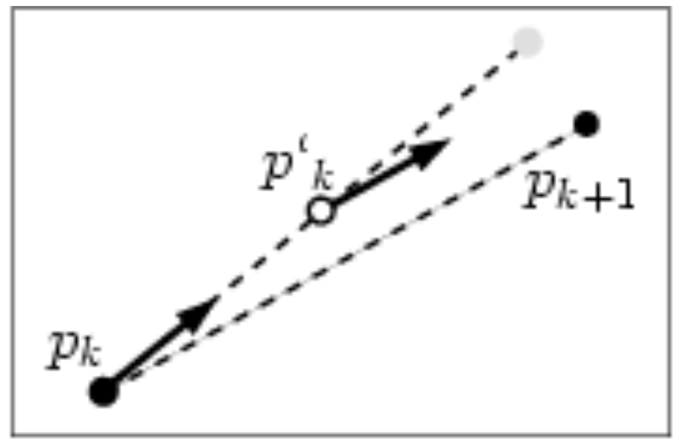
Higher-order methods

$$\int_t^{t+\Delta t} v(L(t))dt$$

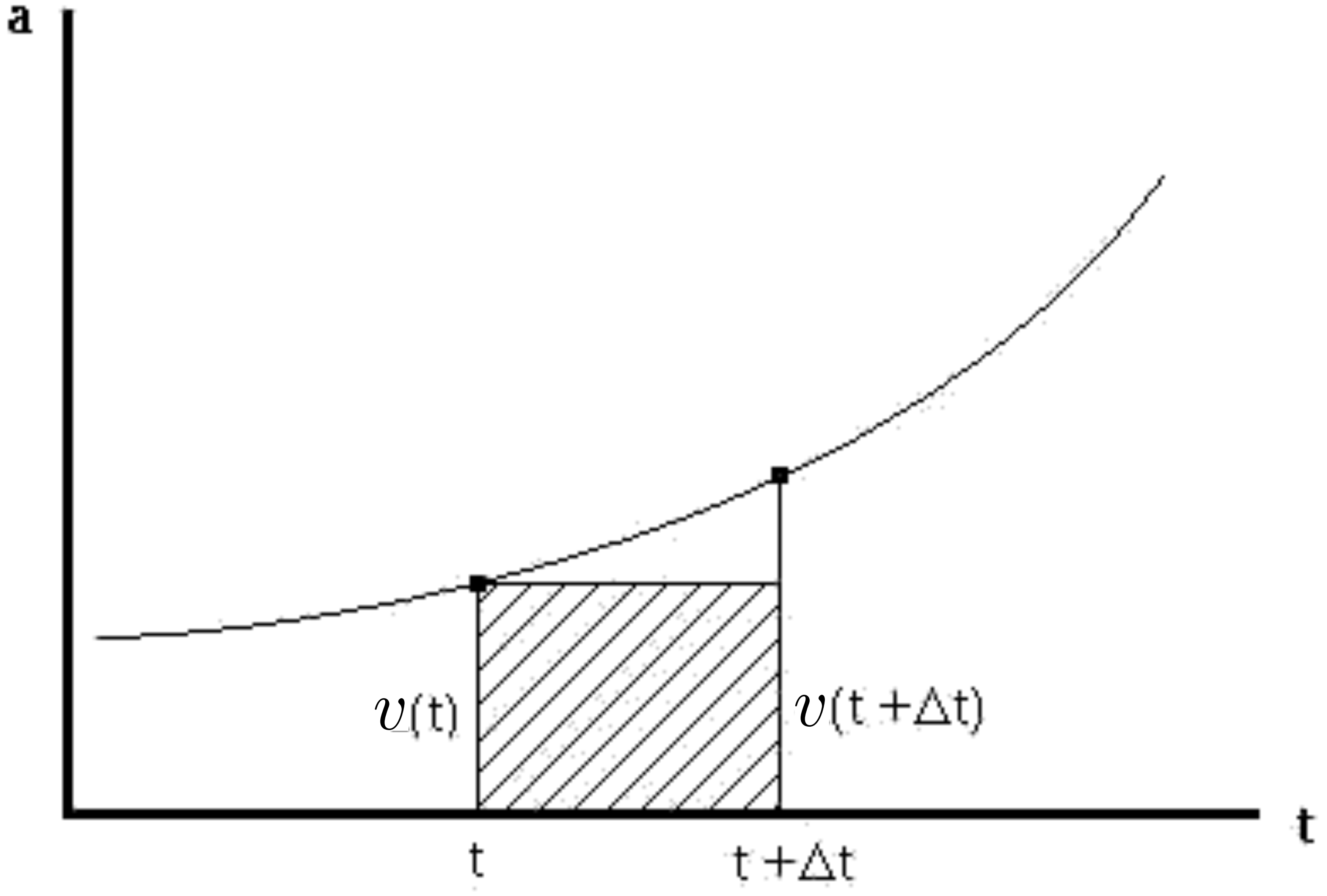
- Euler method (use single sample)



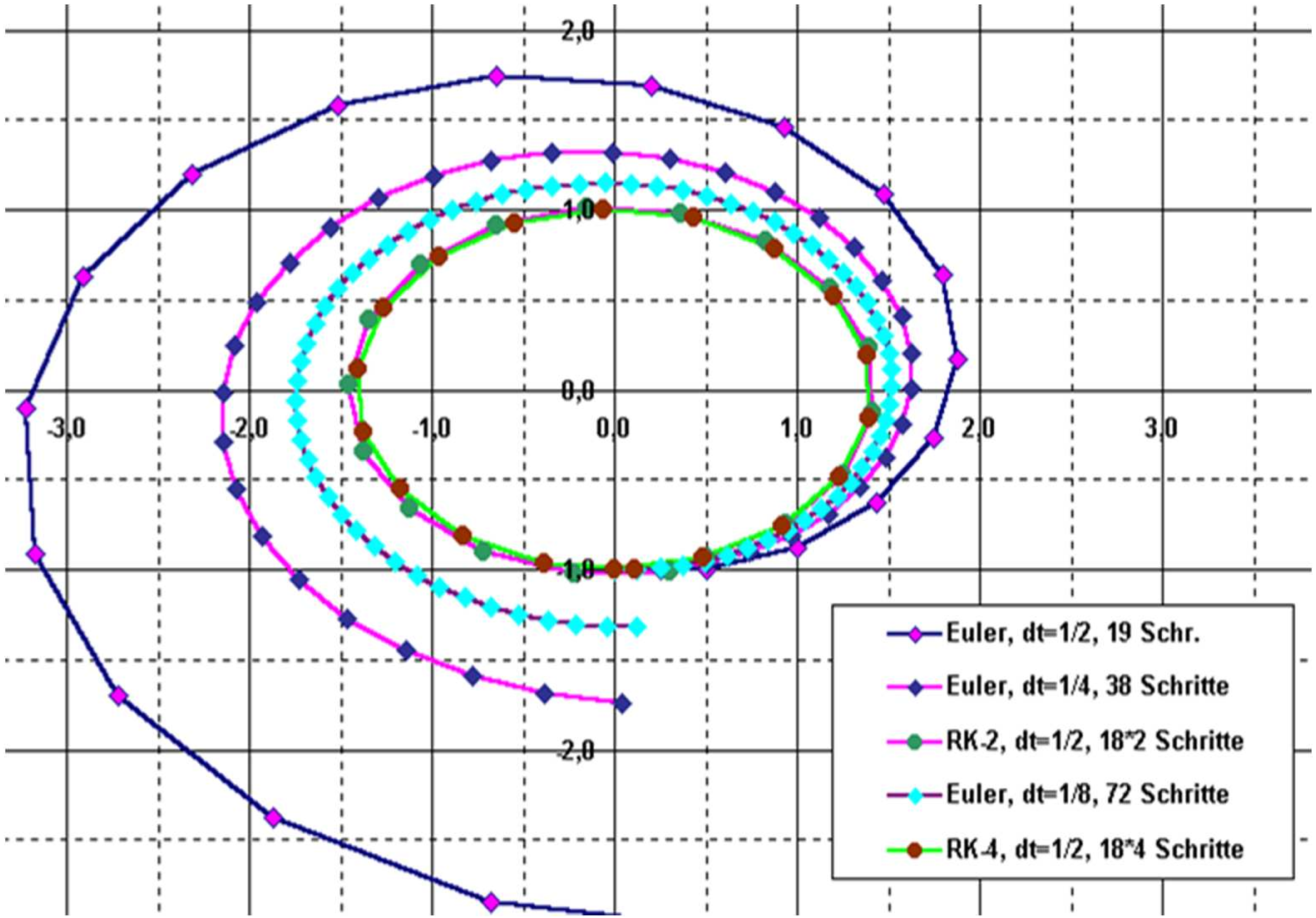
- Higher-order methods (Runge-Kutta) (use more samples)



[A. Mebarki]



Higher-Order Comparison

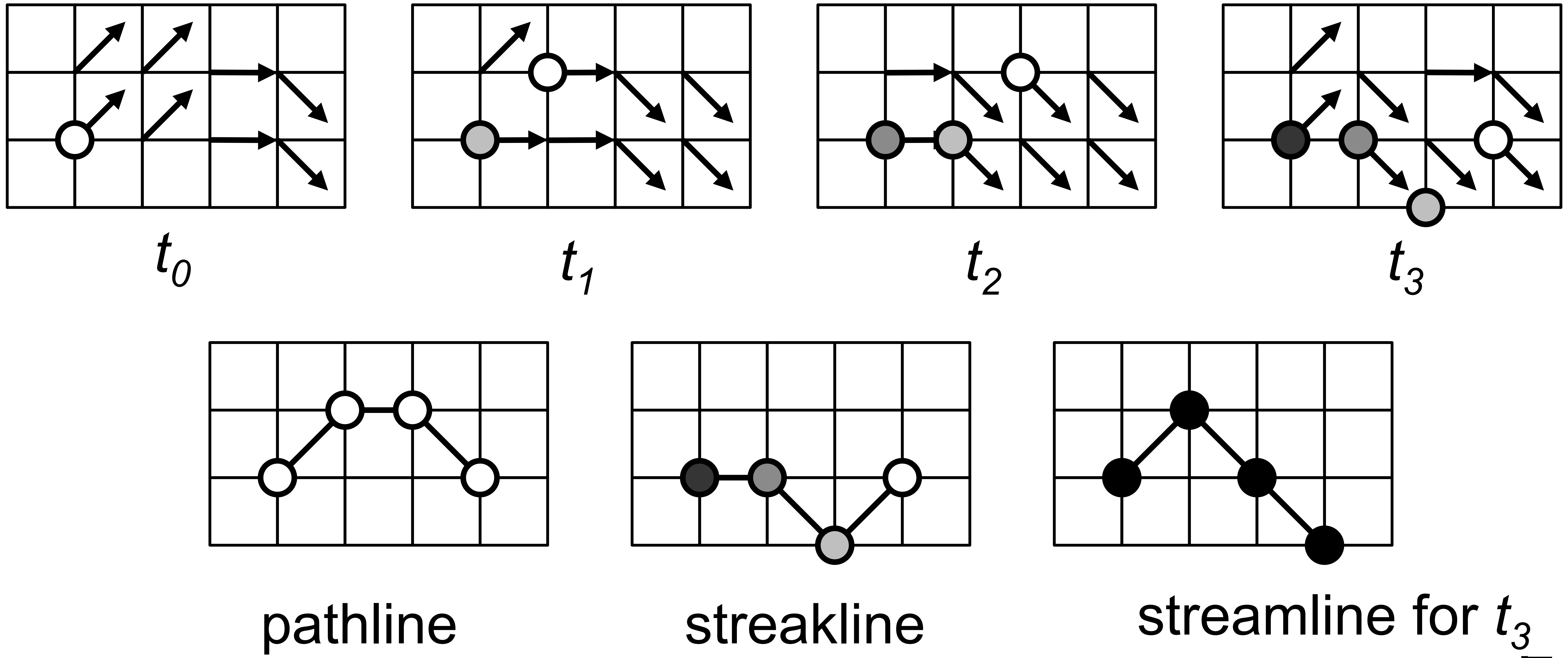


[via Levine]

ParaView Examples

Streamlines & Variants

- Steady vs. **Unsteady** flows
  - In unsteady flows, the vector field **changes** over time
- Variants: **Pathlines** and **Streaklines**



[T. Möller]

Streamlines & Variants

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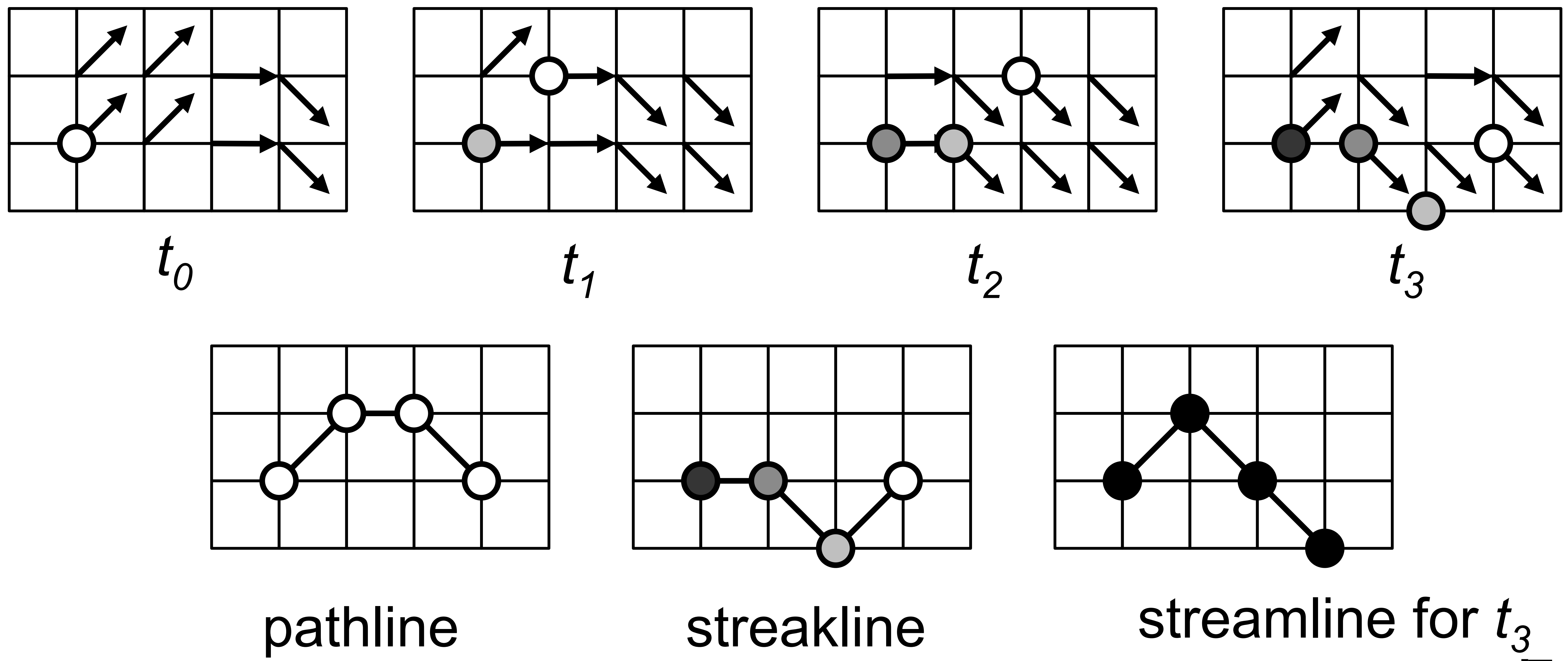
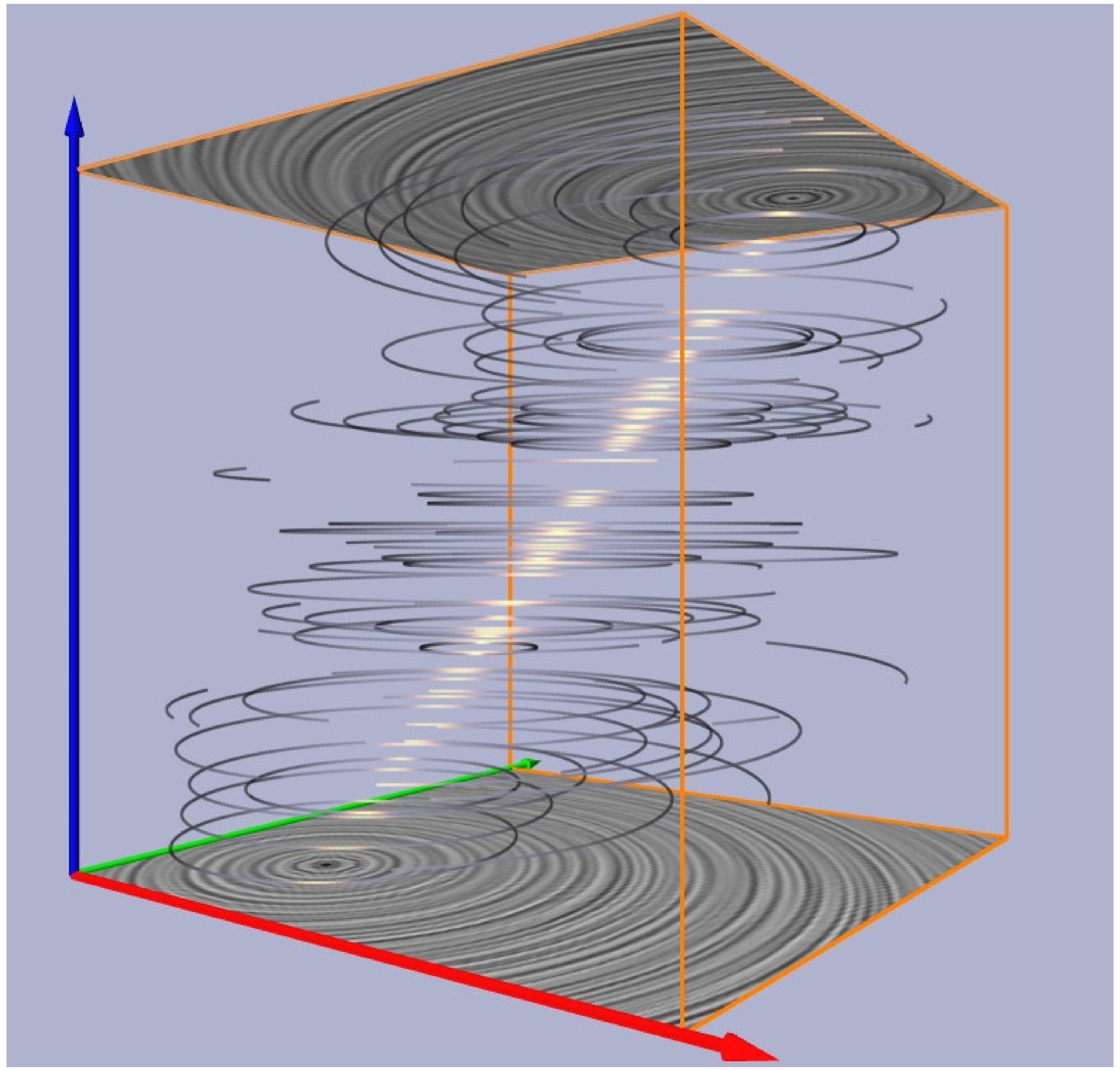


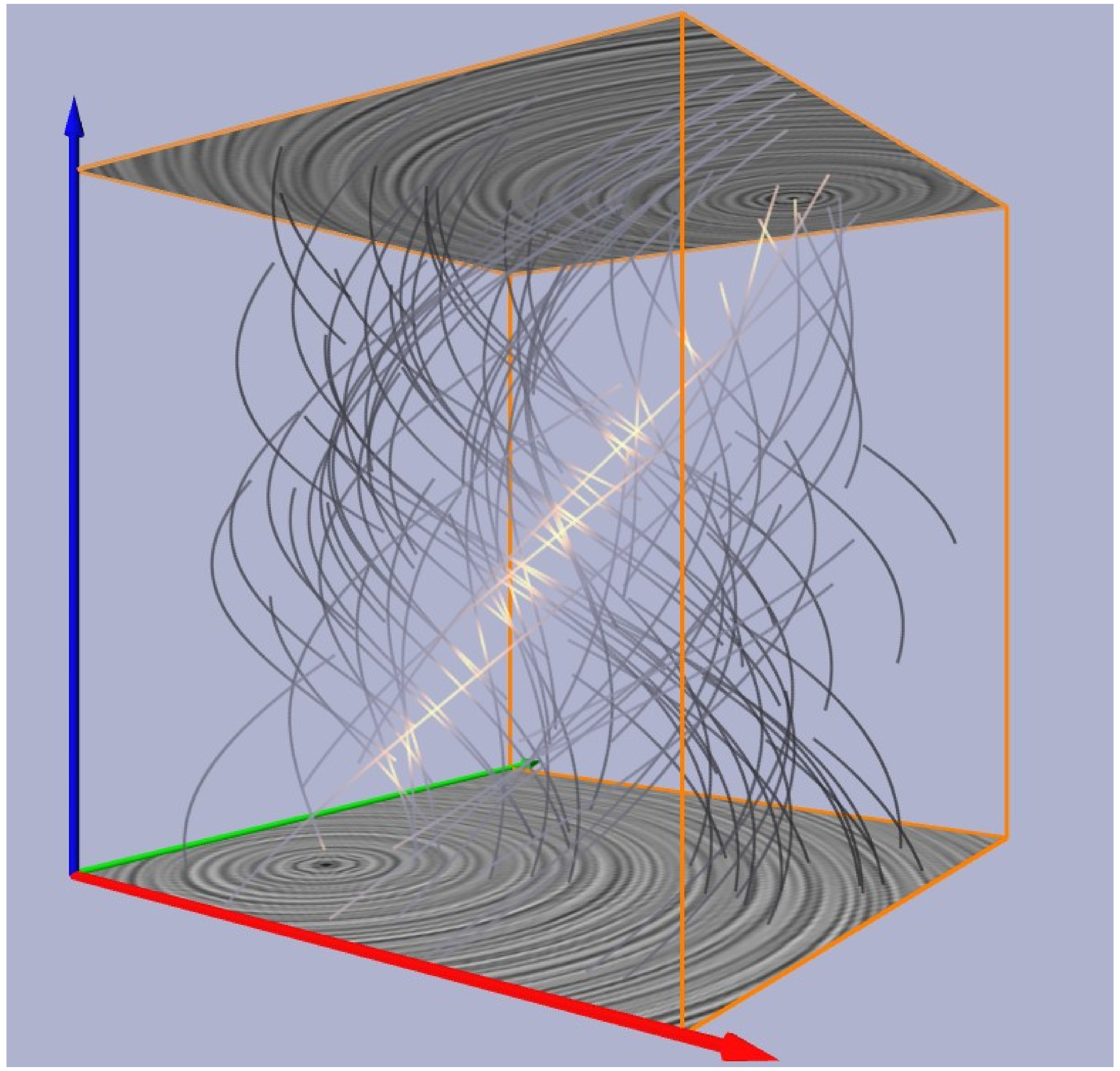
[T. Möller]

All are identical in steady flows!

Streamlines vs. Pathlines



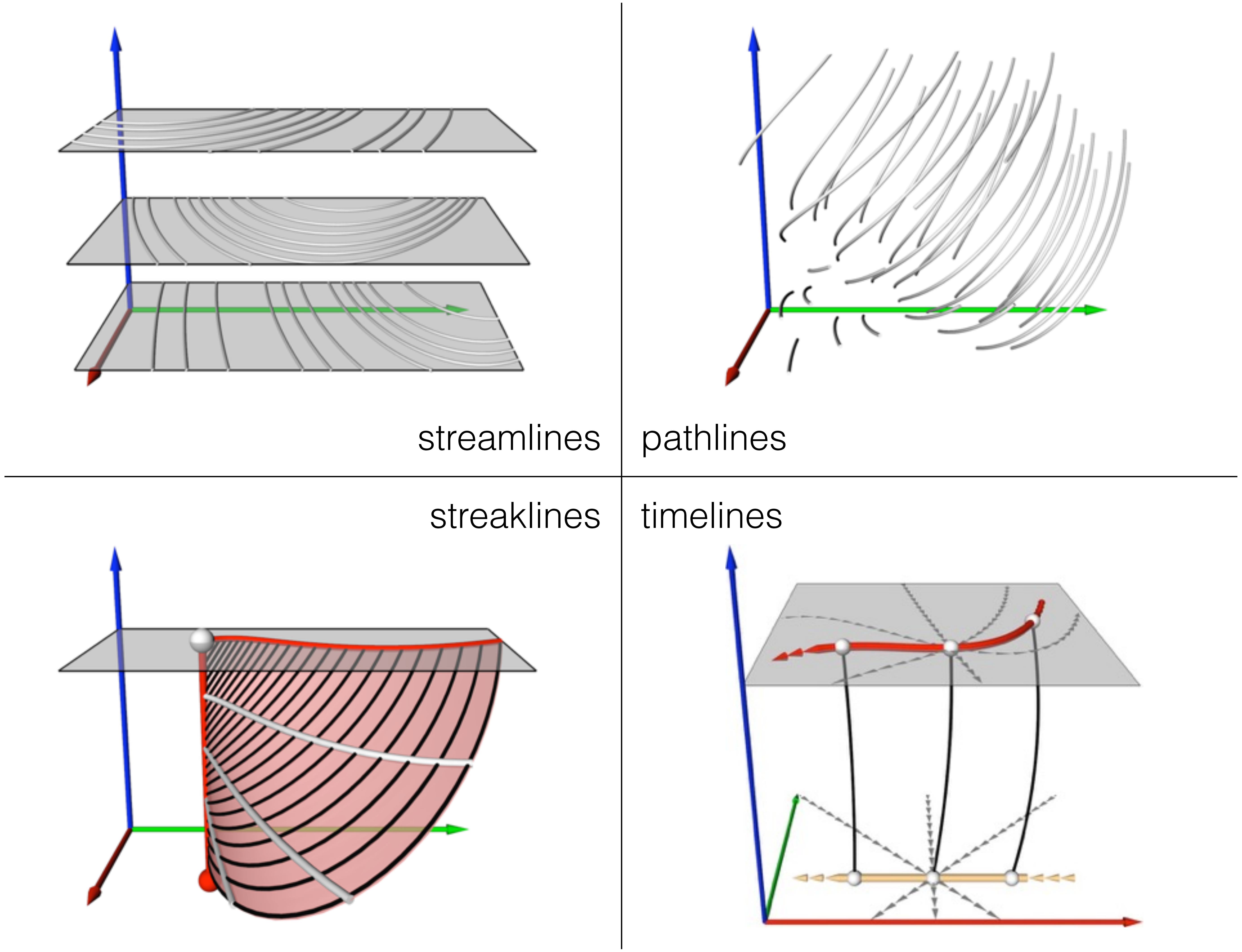
Streamlines



Pathlines

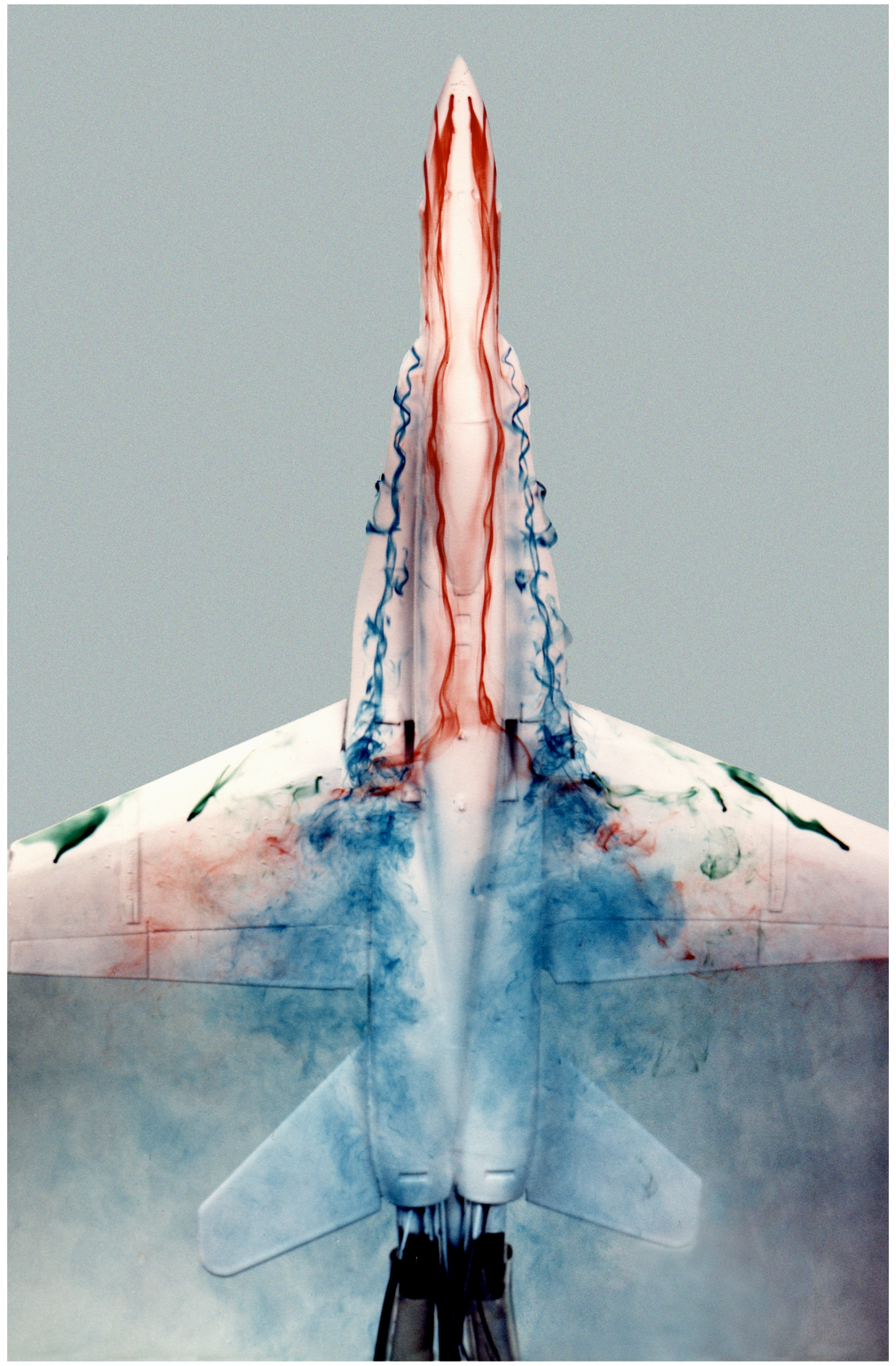
[Weinkauf & Theisel, 2010]

Streaklines and timelines



[via Levine]

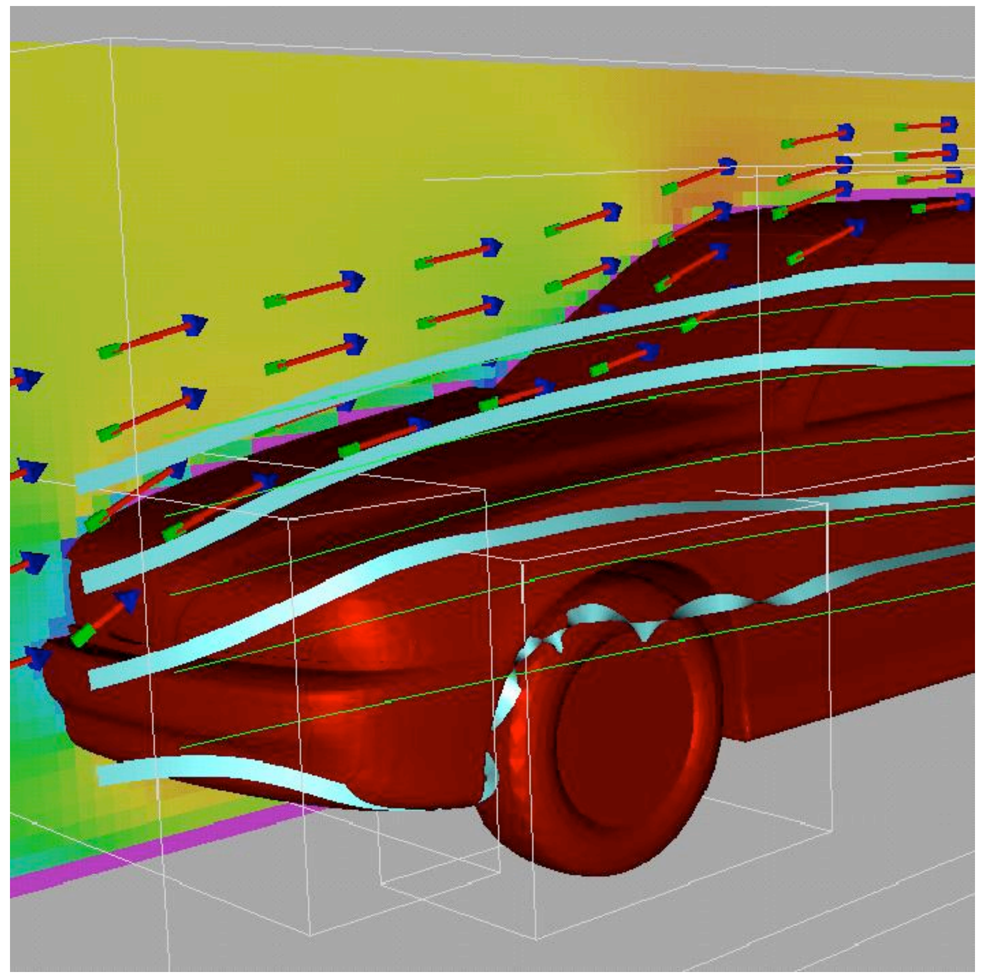
Streamline Variants



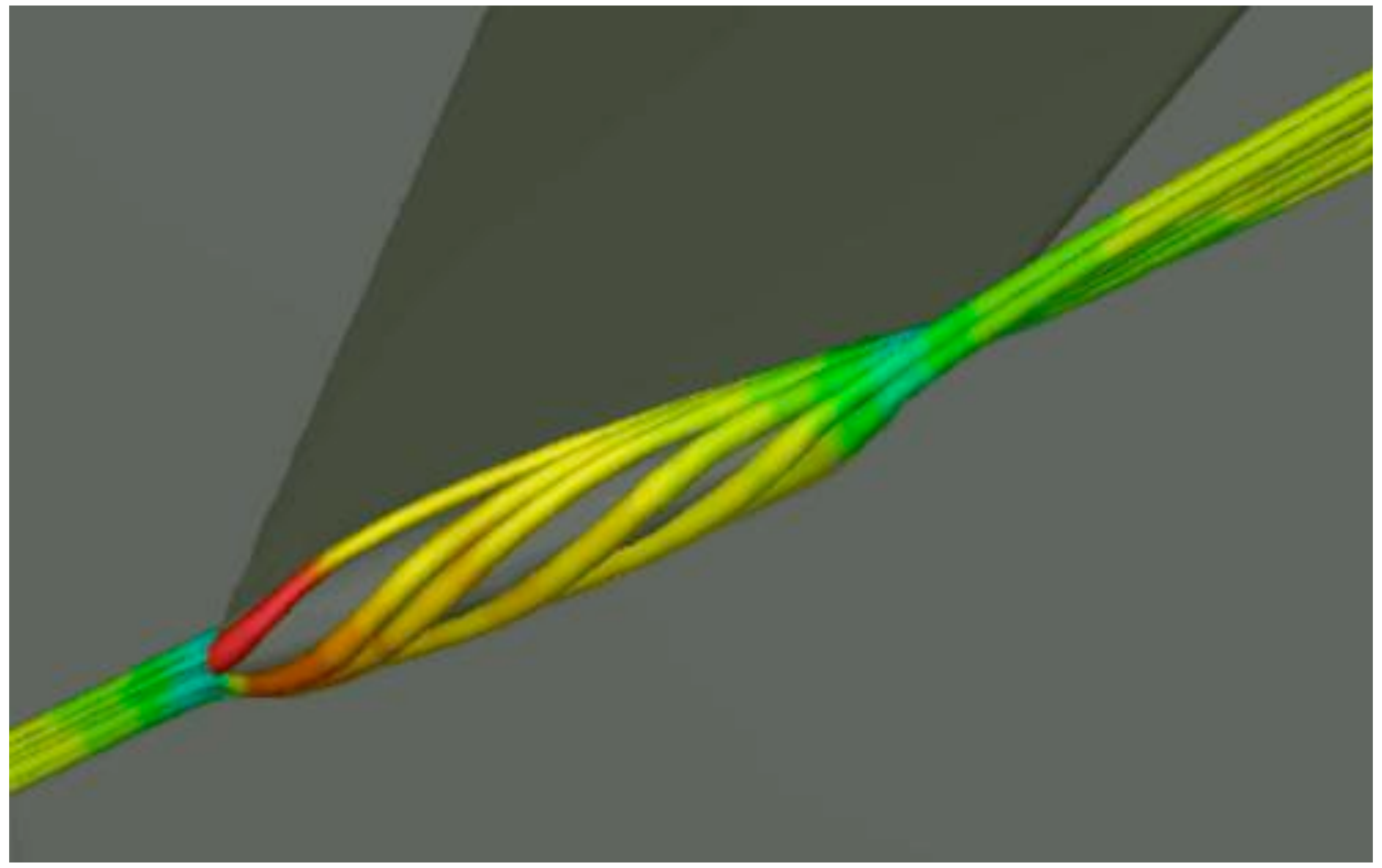
NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: ECN-33298-03 Date: 1985

1/48-scale model of an F-18 aircraft in Flow Visualization Facility (FVF)

Streaklines [NASA]

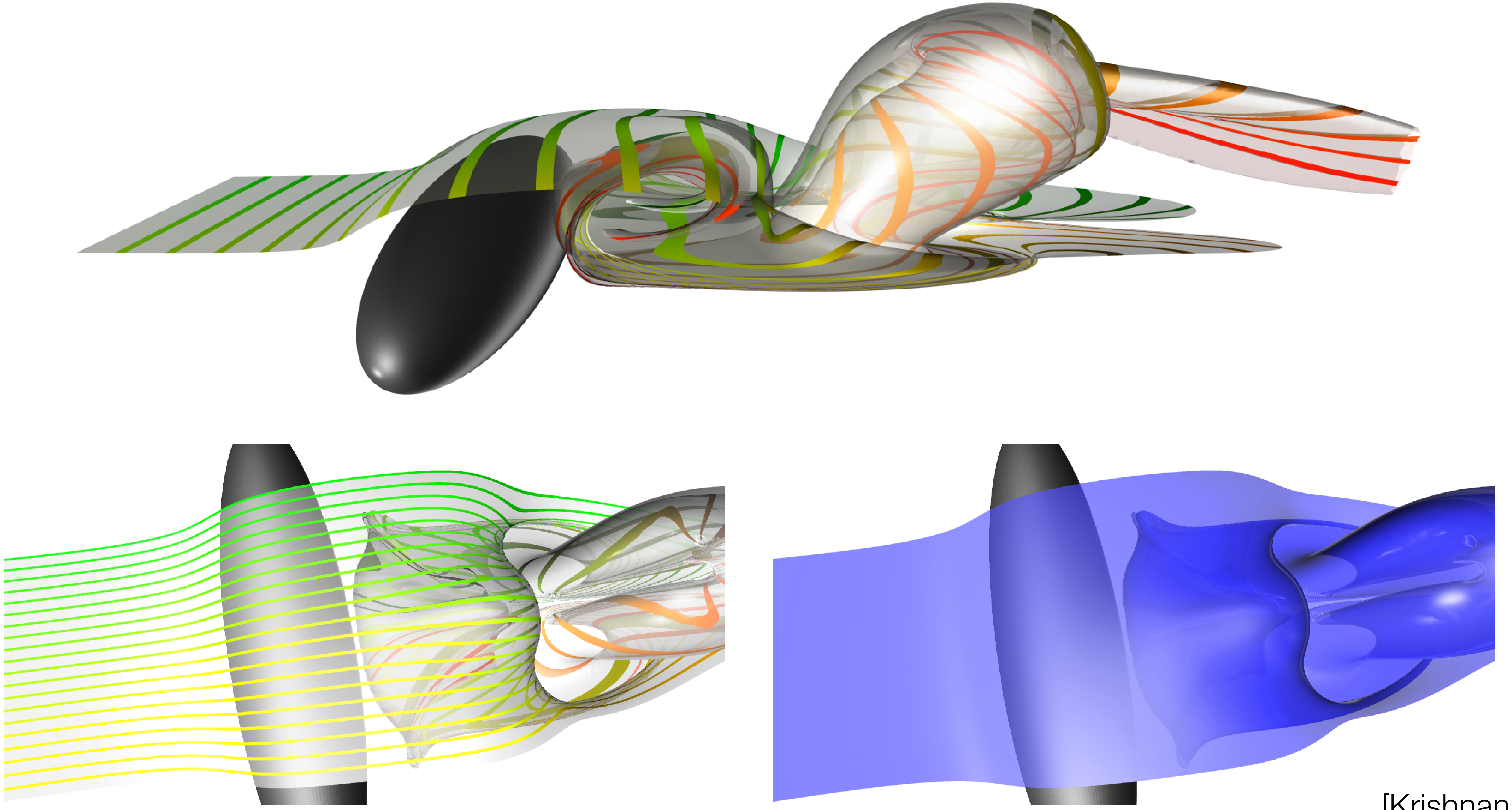


Stream Ribbons [Weiskopf/Machiraju/Möller]



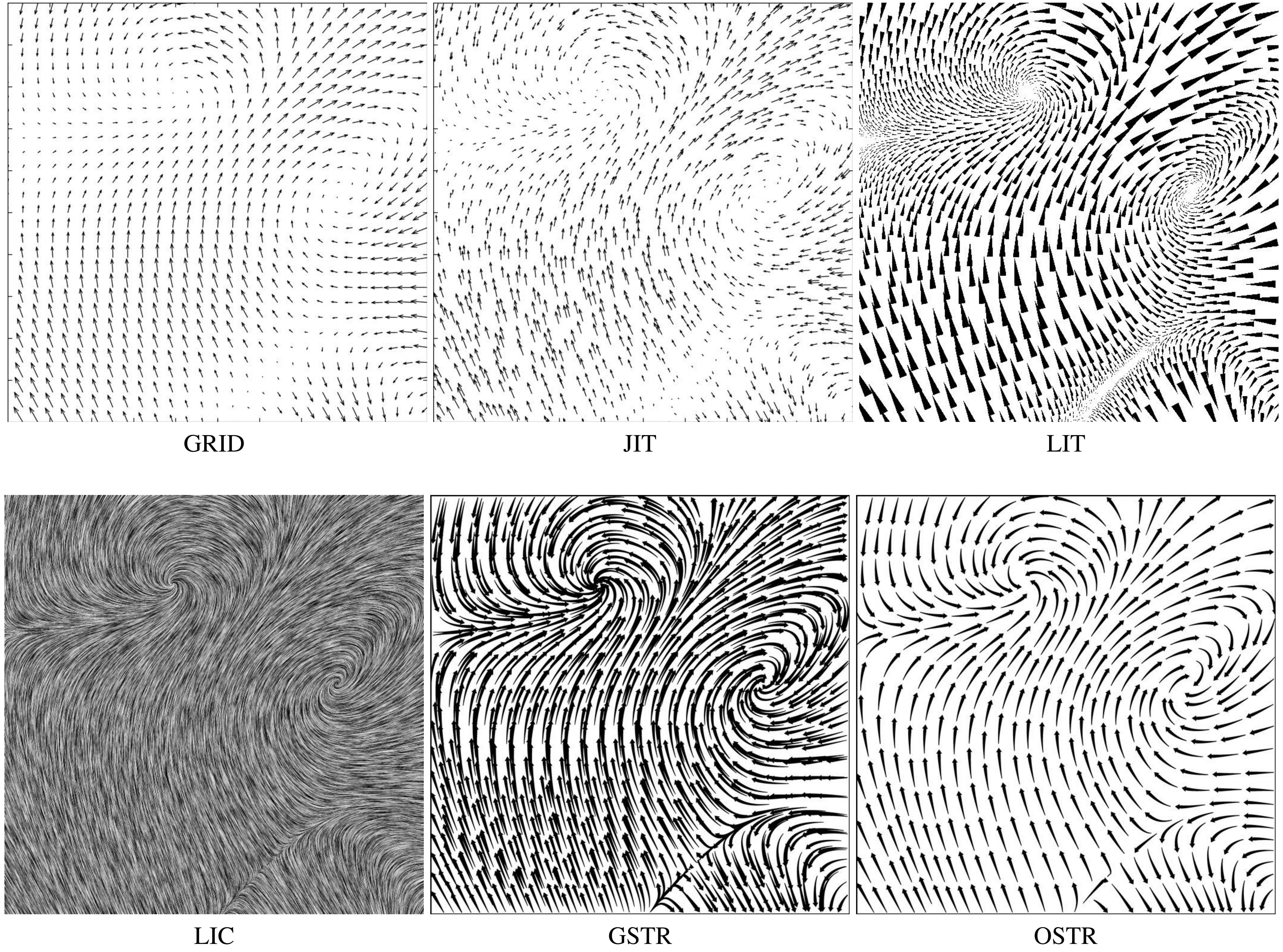
Stream Tubes [Weiskopf/Machiraju/Möller]

Streak Surfaces



[Krishnan et al., 2009]

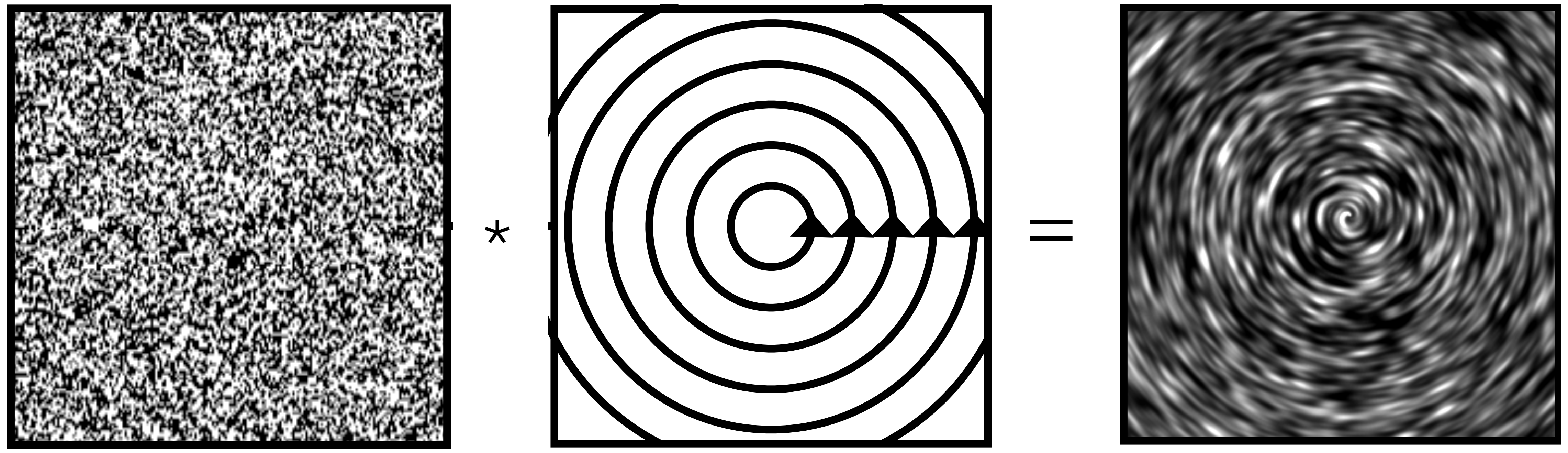
2D Vector Field Visualization Techniques



[Laidlaw et al., 2005]

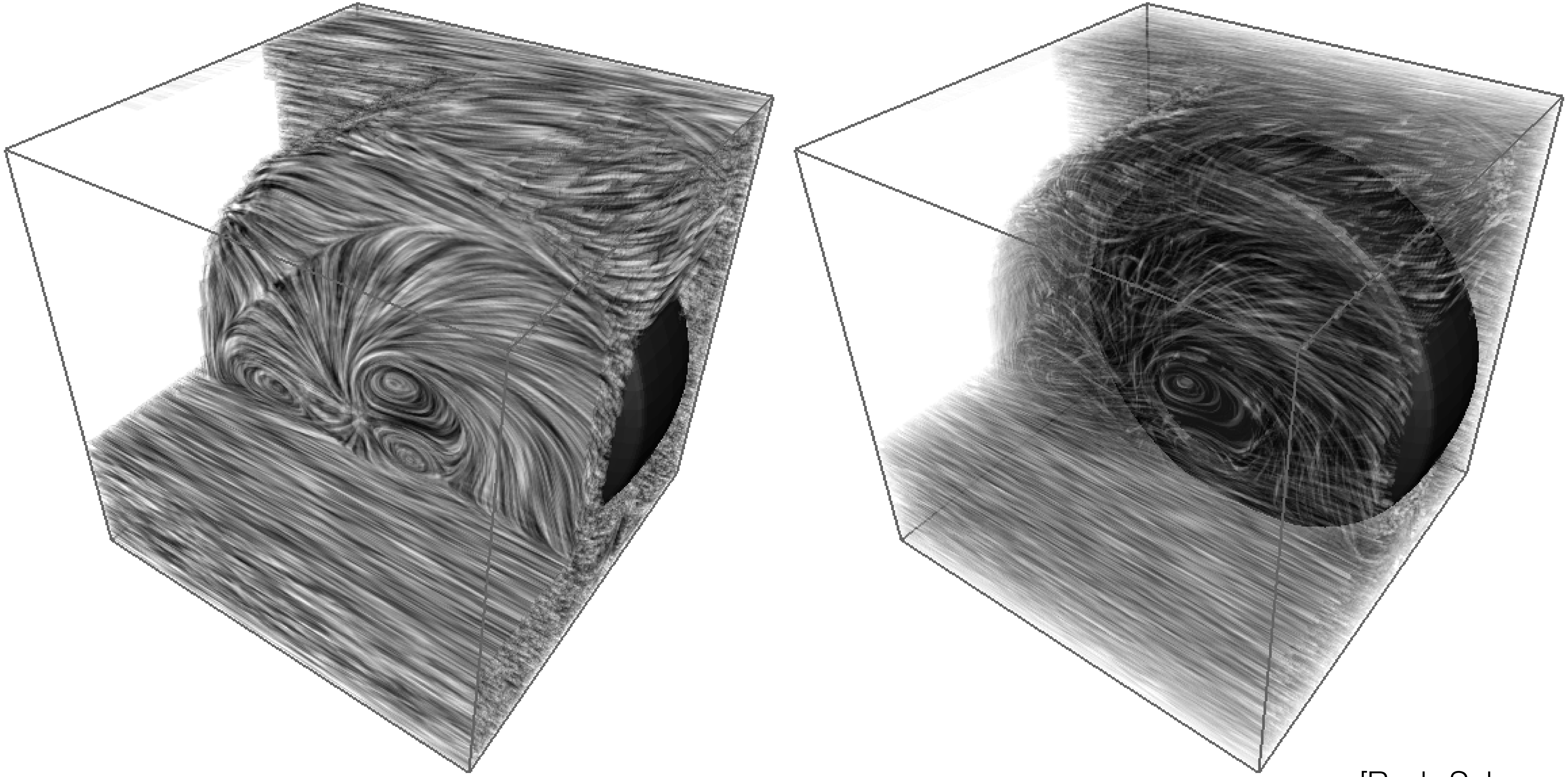
Line Integral Convolution

- Goal: provide a global view of a steady vector field while avoiding issues with clutter, seeds, etc.
- Remember convolution?
- Start with random noise texture
- Smear according to the vector field
- Need structured data



[Weiskopf/Machiraju/Möller]

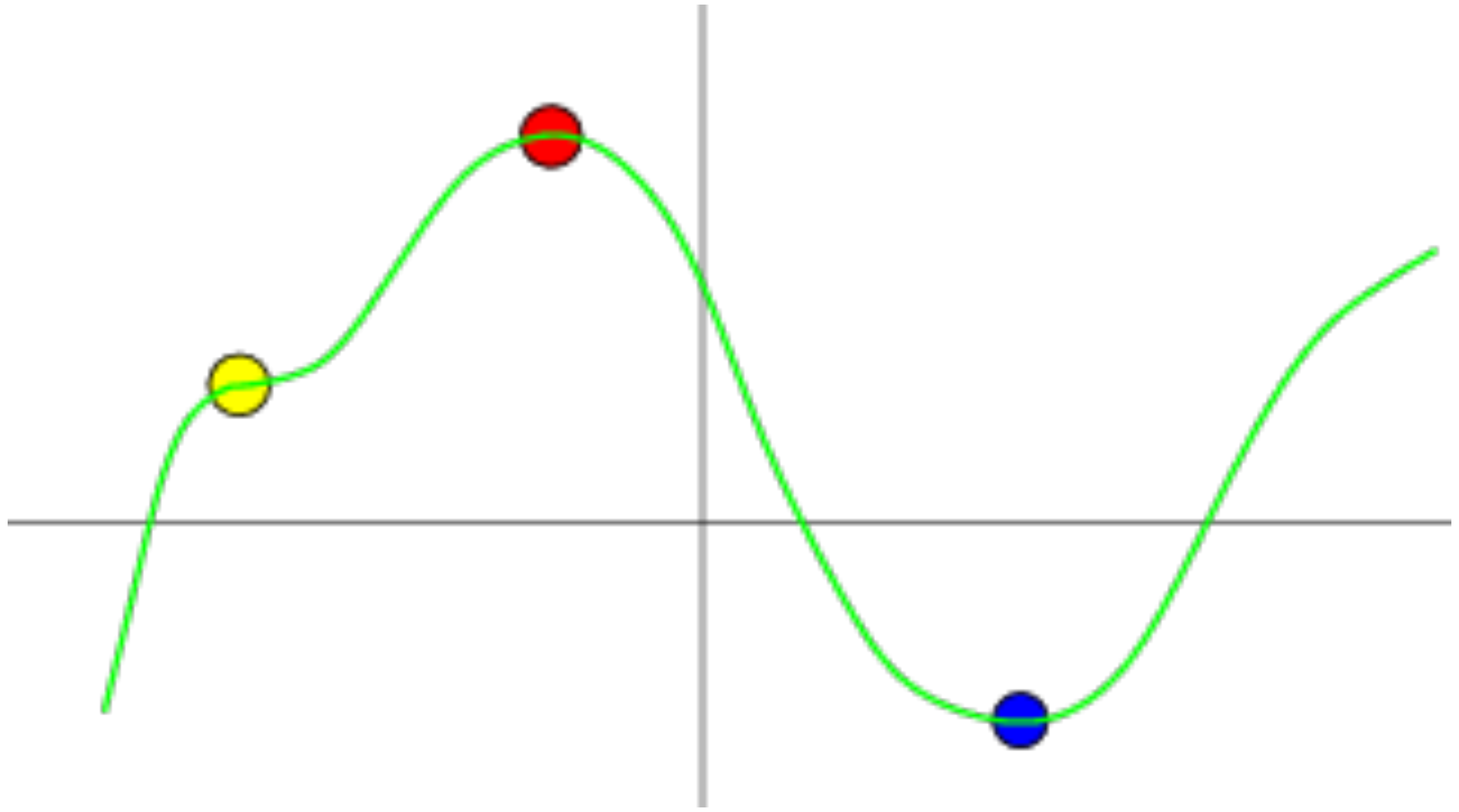
3D LIC



[Rezk-Salama et al., 1999]

Critical Points

- Remember finding min/max for functions?
- Want to understand the general structure of a field, not the exact values
- Find critical points, understand there is a general trend in between
- How?
  - Derivative for functions
  - For fields…gradients

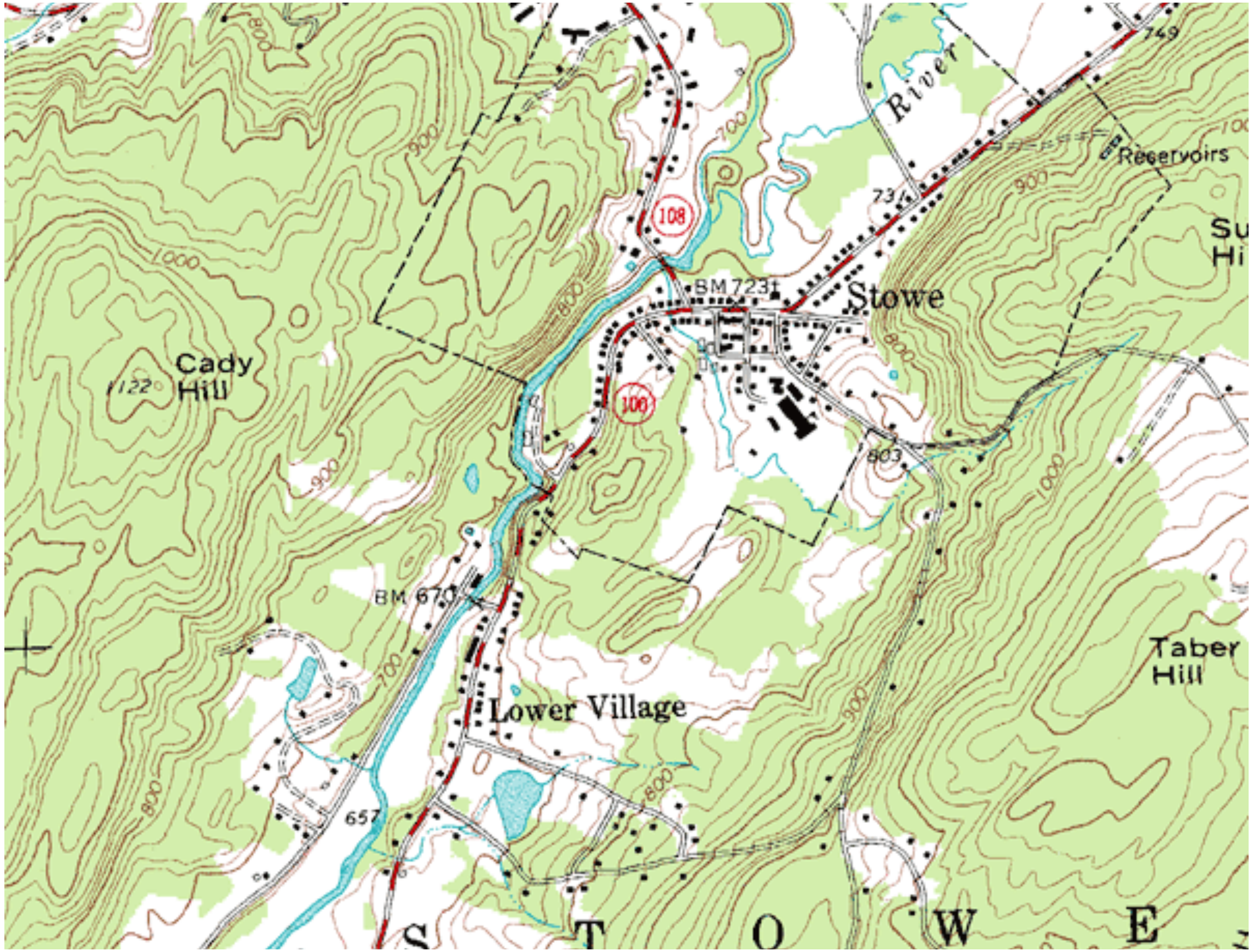


[DQ Nykamp, MathInsight]

Topology

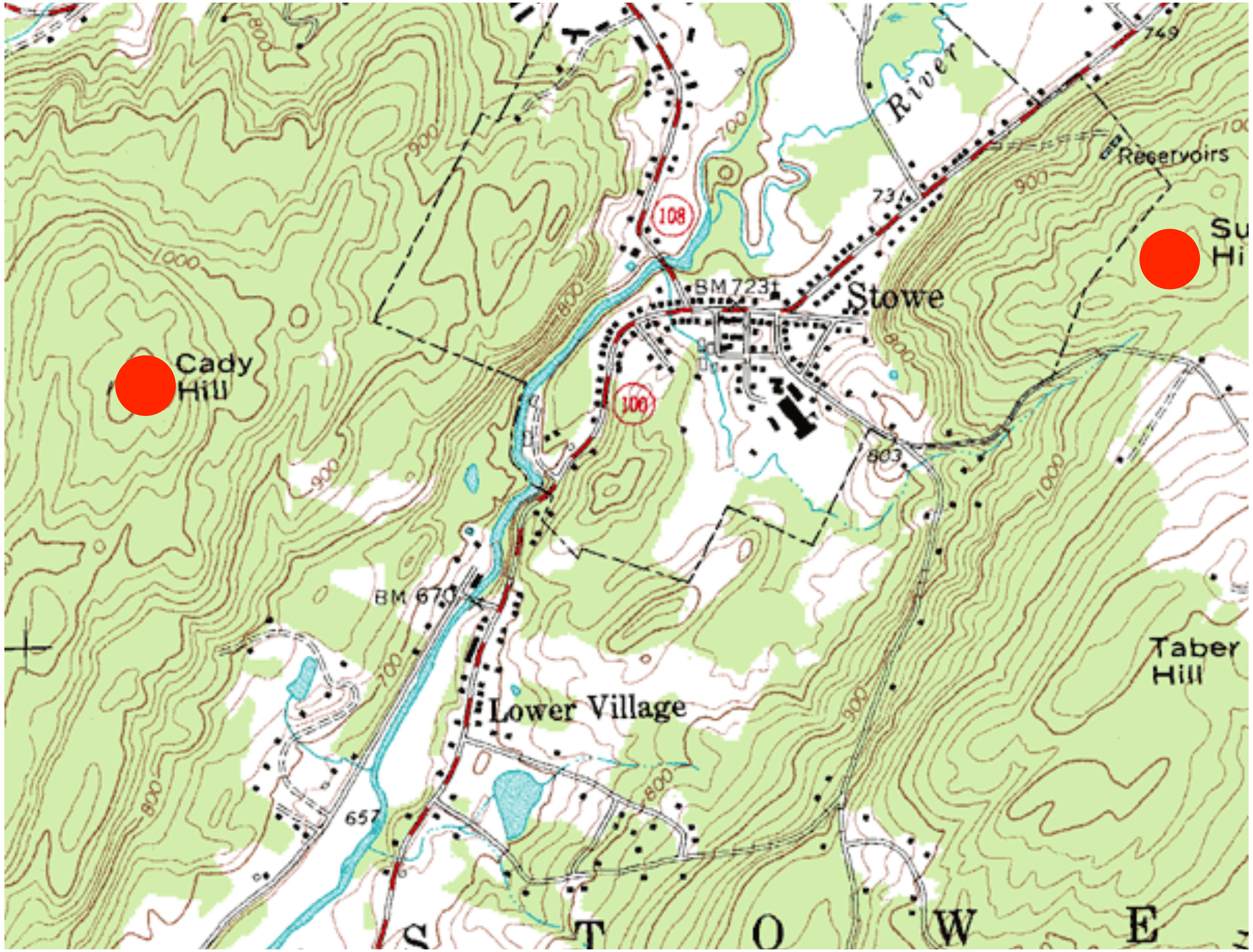
- The general shape of data
- Visualizations that can be "stretched" to resemble each other are topologically equivalent
- Technically, continuous transformations don't change anything
- Connect critical points to obtain a general picture of the data
- Can talk about topology in both scalar and vector fields

2D Scalar Field Topology



[Wikipedia]

2D Scalar Field Topology

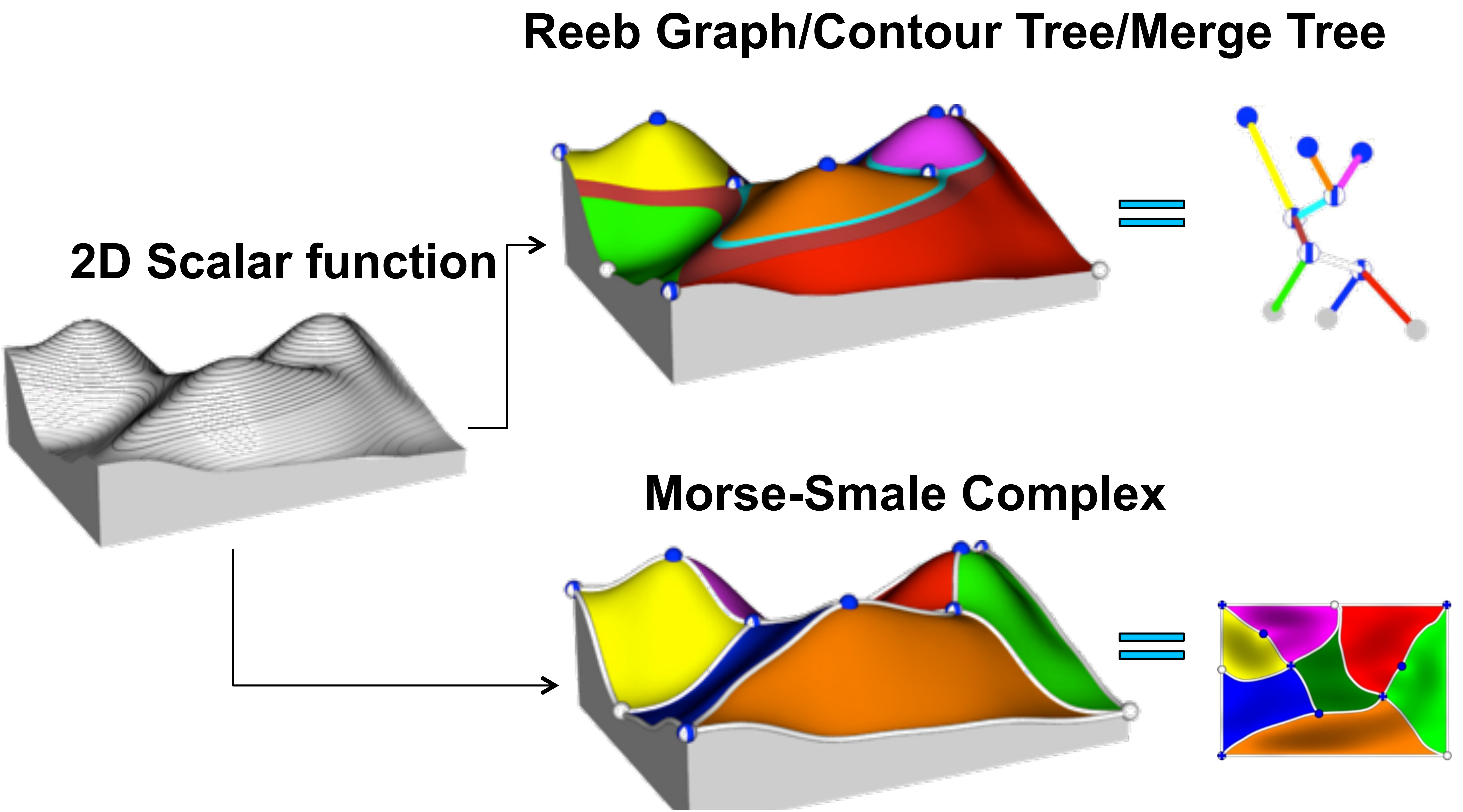


[Wikipedia]

Scalar Field Topology

- Examine the gradient (changes between points on the grid) of the scalar field
- Where the gradient is zero, we have critical points (max, min, saddle)
- Can build Reeb Graph, Contour Tree, or Morse-Smale Complex from this information to show the topology (with some reasonable assumptions about how the scalar field looks)

Scalar Field Topology



[via Levine]

Vector Field Topology

- Instead of "guessing" correct seed points for streamlines to understand the field, try to identify structure (topology) of the field

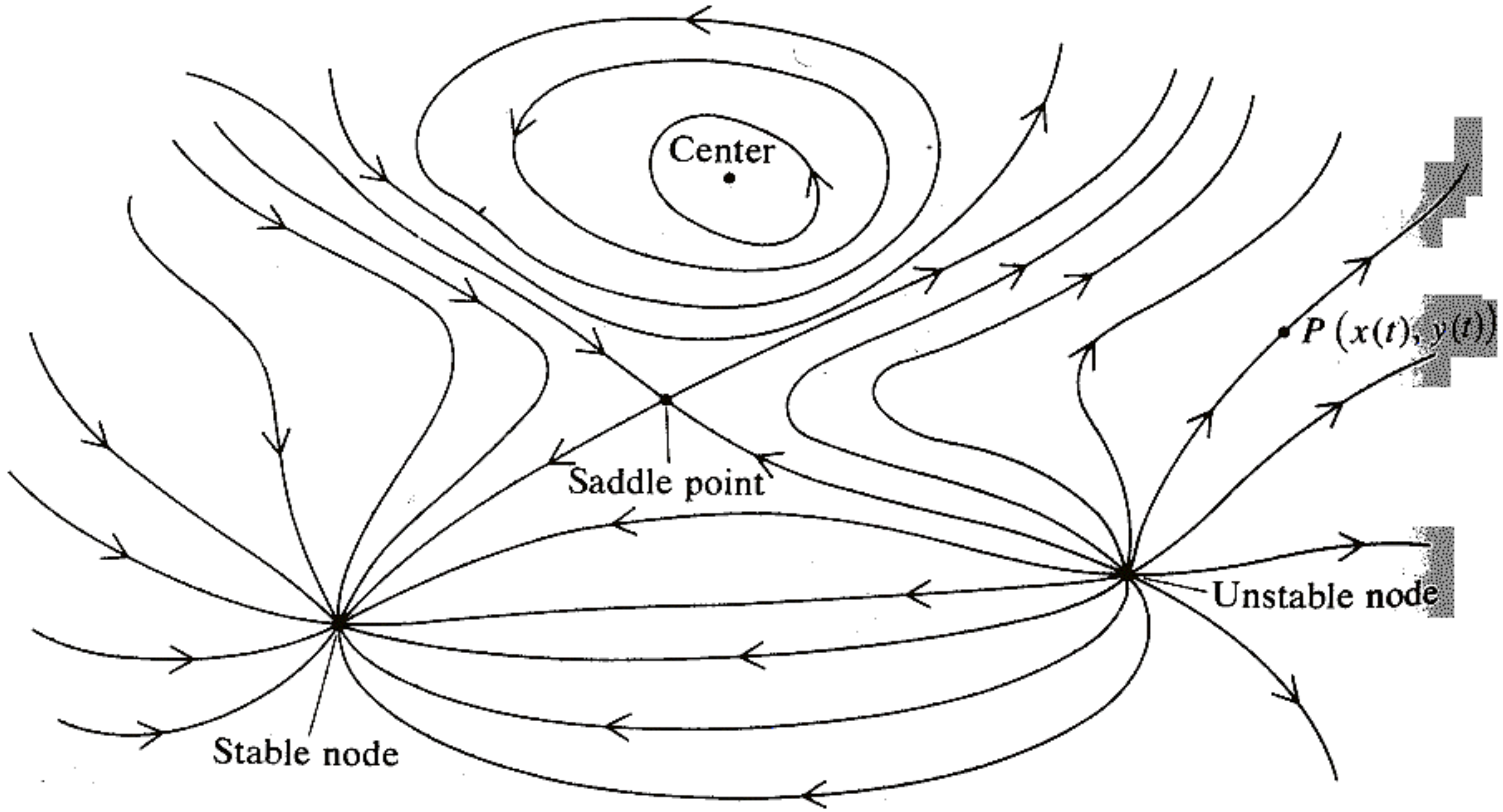
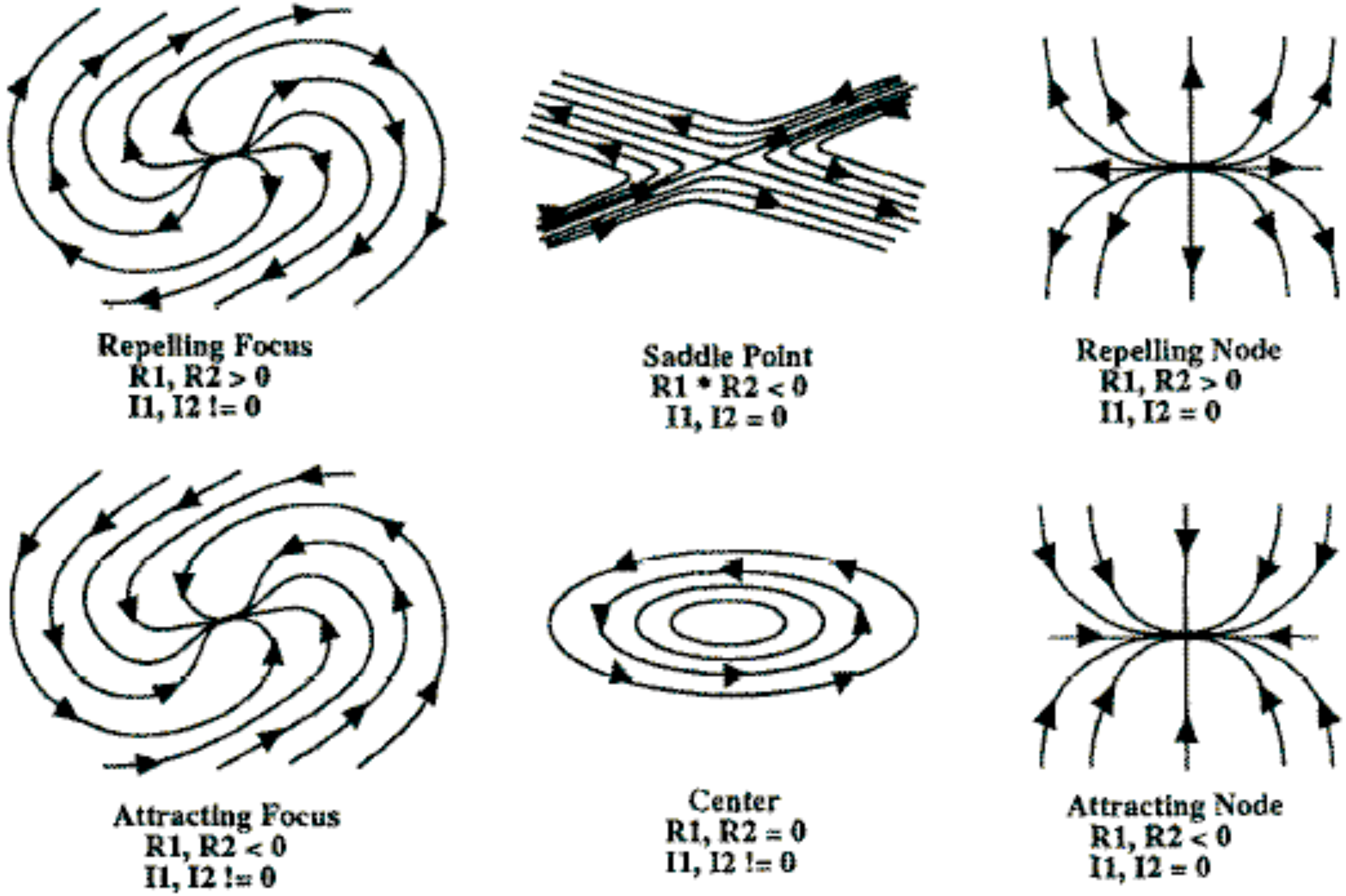


Figure 7.1 A phase portrait.

[M. Henle]

Critical Points

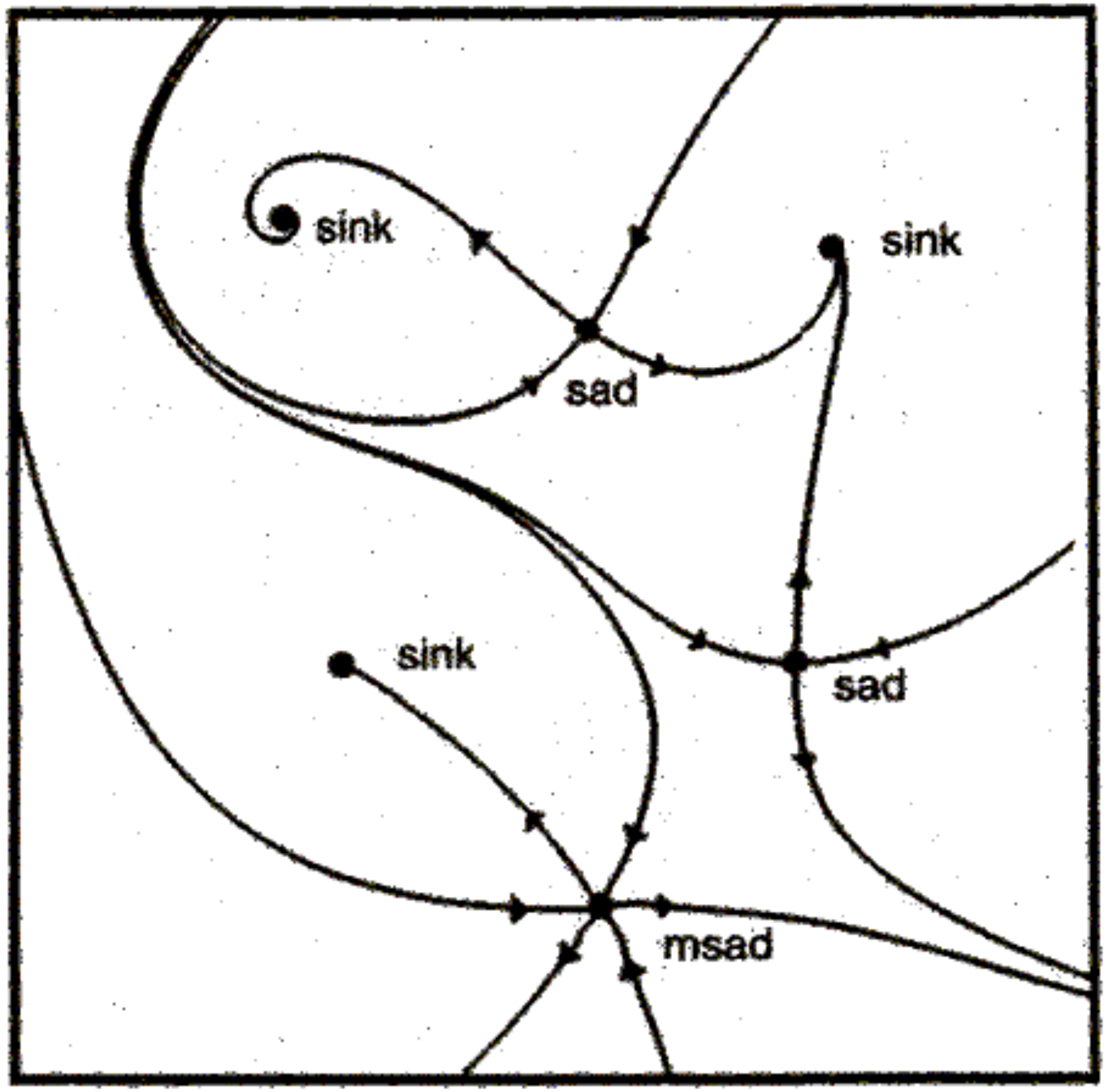


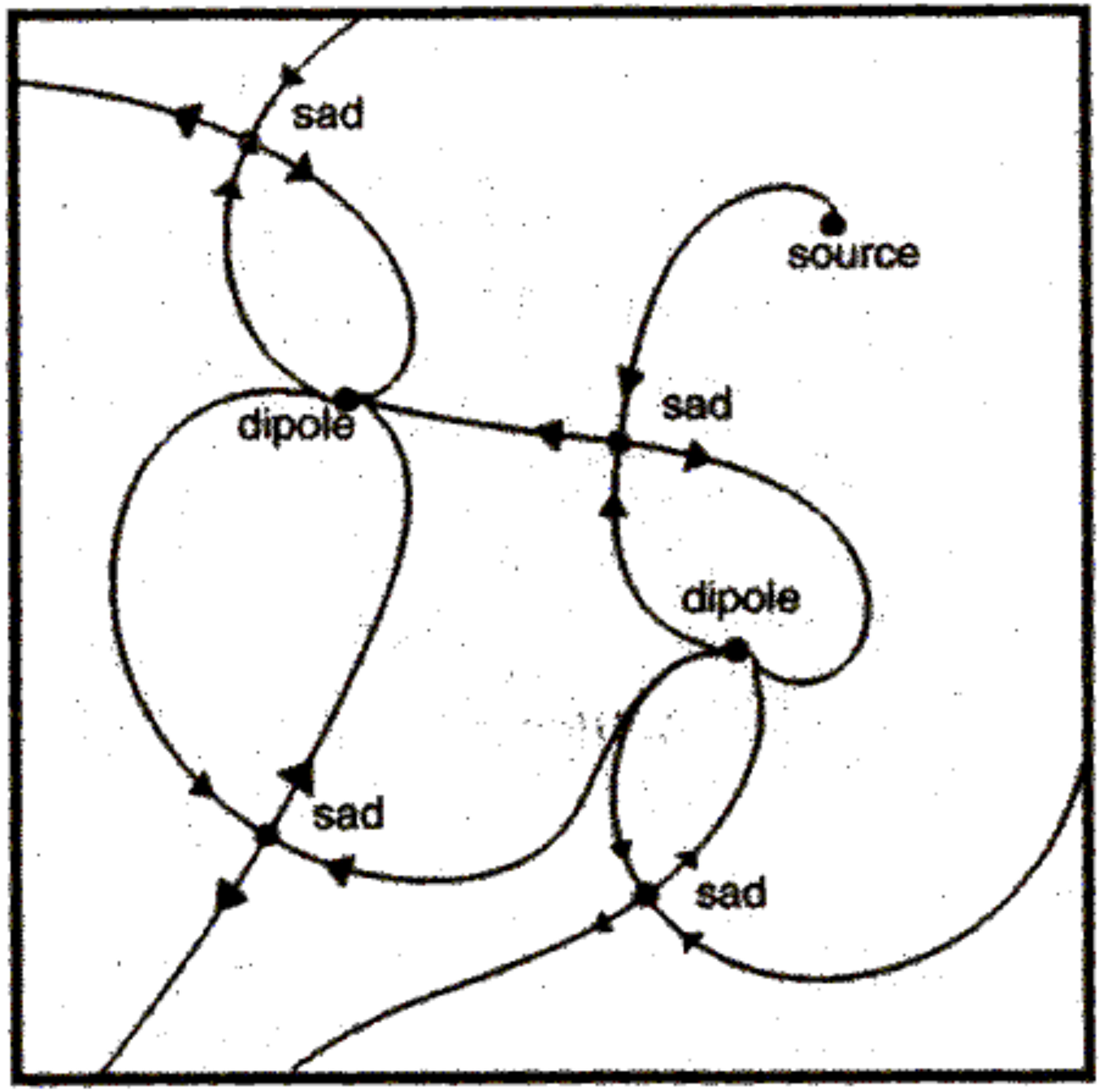
[Helman & Hesselink]

Critical Points

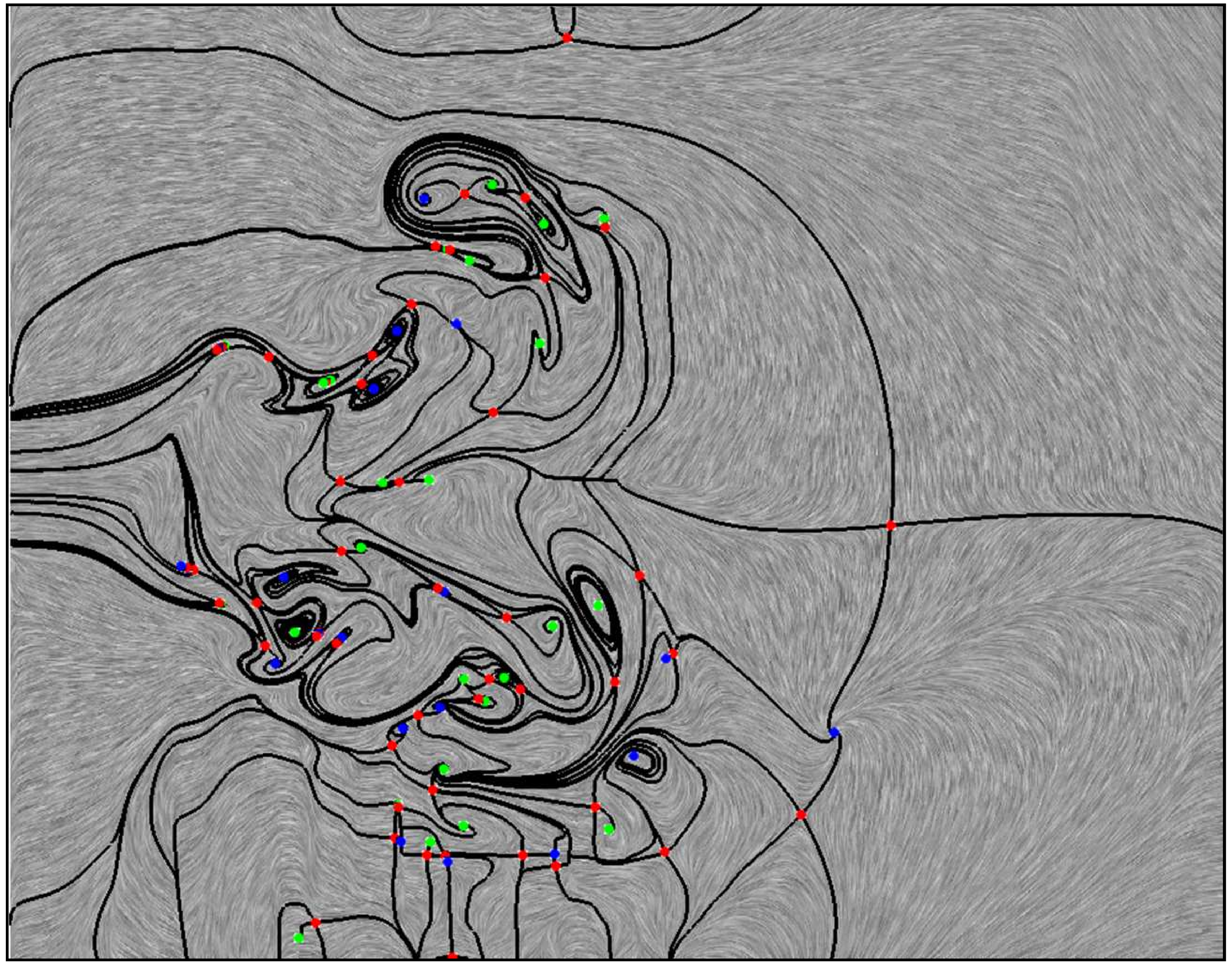
- Critical Points
  - Find where the vector field vanishes (the zero vector or undefined)
  - Attracting Nodes (Sinks), Repelling Nodes (Sources), Attracting Foci, Repelling Foci, Saddles, Centers
- How to find such points?
  - Can use a similar idea to Marching Cubes
  - Use the eigenvalues of the Jacobian matrix to classify

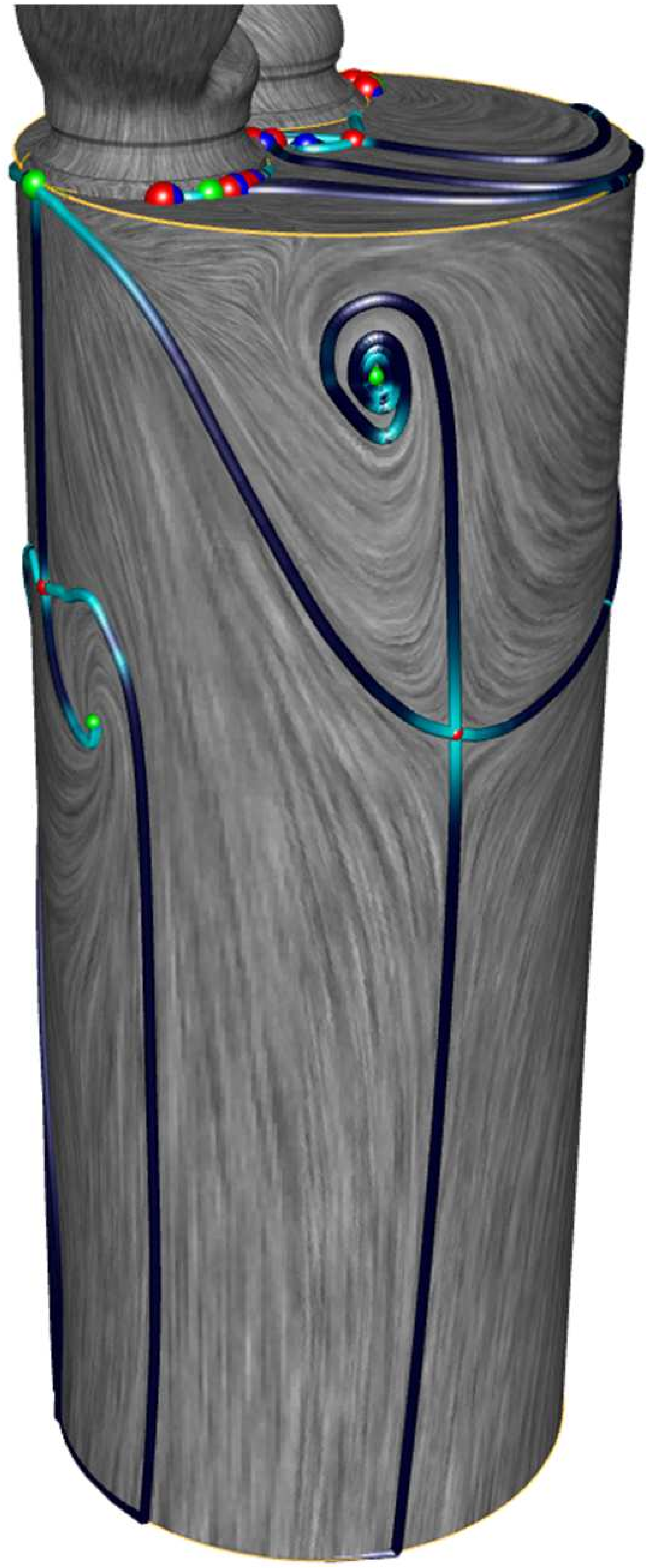
Topological Skeleton





More Examples





[Levine]

Course Evaluations